

Pointer's Gamut, MacAdam Limits, and Wide-Gamut Displays

Phil Service
Flagstaff, Arizona, USA
23 May 2016

Summary

Three "wide-gamut" RGB color spaces — AdobeRGB, DCI-P3, and Rec. 2020 — are compared to the gamut of real surface colors (Pointer's gamut) and the optimal color solid (MacAdam limits). Only Rec. 2020 covers Pointer's gamut completely, or very nearly so, at all luminance levels examined. Rec. 2020 also approaches or exceeds the MacAdam limits in all color regions at high luminance ($L^* \geq 80$). Better coverage of the MacAdam limits at lower luminance levels will most likely require multi-primary displays.

Key words: Pointer's gamut, MacAdam limits, surface colors, optimal colors, AdobeRGB, DCI-P3, Rec. 2020, wide-gamut displays, Rec. 2020-to-*XYZ* conversion matrix, DCI-P3-to-*XYZ* conversion matrix.

1. Introduction

1.1 Optimal Colors and MacAdam Limits

Almost everything we can see, we see by reflected light. An object has color because it selectively absorbs some wavelengths of light, and reflects others. In 1917, the German chemist Wilhelm Ostwald proposed that the most saturated reflective colors possible have a very special spectrum: at any given wavelength, they are either complete absorbers or complete reflectors. Furthermore, across the visible spectrum, there can be at most only two transitions between the

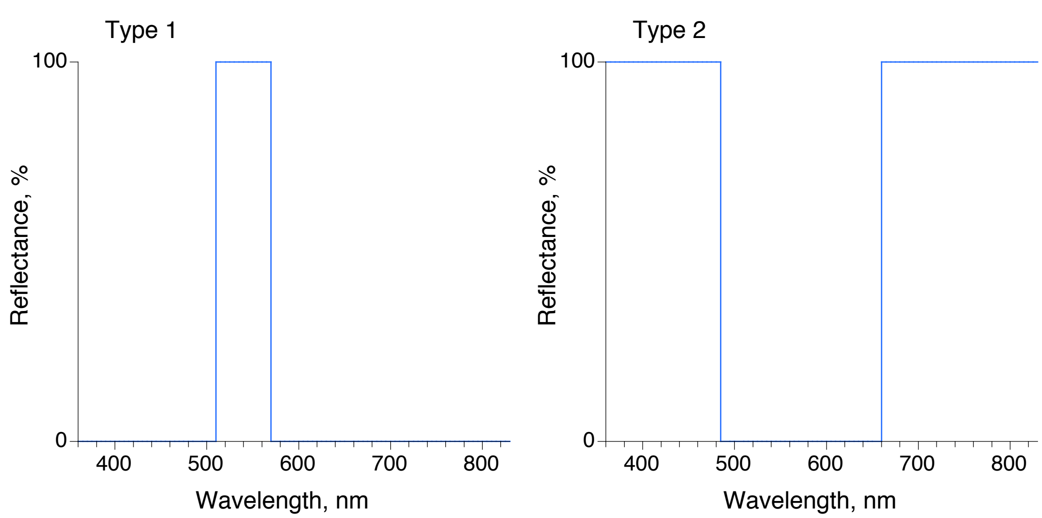


Fig. 1. Examples of the two spectral reflectance patterns of optimal colors. Left: Type1; Right: Type 2

two states: spectral reflectance can go from zero (perfect absorbance), to one (perfect reflectance), and back to zero (Type 1); or it can go from complete reflectance, to zero reflectance, and back to complete reflectance (Type 2, Fig. 1). Colors produced by either of these spectral reflective patterns are generally referred to as *optimal colors* — they are the most saturated (chromatic) colors possible for a given luminance.

Erwin Schrödinger provided a logical confirmation of Ostwald's hypothesis in 1920. David MacAdam (1935a) gave a proof based on the CIE 1931 *XYZ* color matching functions. MacAdam also calculated the chromaticity coordinates of the optimal color solid with sufficient completeness to plot them on the CIE 1931 *x-y* chromaticity diagram (1935b). Because these coordinates represent the most saturated possible surface (reflective) colors, they are now commonly referred to as *MacAdam limits.* The limits define the loci of optimal colors. They depend upon luminance, or lightness; and on the spectral power distribution of the reference illuminant — for example, D65.[1]

It needs to be emphasized that the concept of optimal color applies only to surfaces that are seen by reflected light, or to colors produced by filters (transmissive light). Fluorescent colors are specifically excluded; as, presumably, are glossy colors that can be specular reflectors. As a corollary, it is entirely possible for *lights* to produce colors that are more saturated than any optimal surface color: for example, any reasonably bright monochromatic light source.

1.2. Pointer's Gamut

In 1980, Michael Pointer published the results of a colorimetric analysis of over 4,000 natural and man-made surface colors (Pointer 1980). Whereas the MacAdam limits set a theoretical maximum, Pointer's data is an attempt to establish the empirical limits of color saturation. More recently, the Standard Object Colour Spectra database for color reproduction evaluation (SOCS) has been developed as an ISO standard. It includes spectral information on more than 50,000 colors. The SOCS gamut does not differ greatly from Pointer's gamut, and will not be considered further in this paper.[2]

1.3 Wide-Gamut Displays

Displays that can reproduce all, or almost all, of the AdobeRGB color space are generally referred to as wide-gamut displays. Currently, such displays are available from a number of manufacturers. In addition, the 2015 27-inch Retina iMac and the 2016 9.7-inch iPad Pro have displays that can reproduce the DCI-P3 gamut. DCI-P3 is a standard developed by the Digital Cinema Initiatives organization and published by the Society of Motion Picture and Television Engineers. It is also referred to as a wide-gamut color space, and is used for digital movie

---

[1] Ostwald received the Nobel Chemistry Prize in 1909. Schrödinger received the Nobel Physics Prize in 1933 (for his work on quantum mechanics). MacAdam (1910 – 1998) didn't get a Nobel Prize, but presumably had the satisfaction of seeing his name applied to the theory of optimal colors. A short biography can be found here.

[2] Inui, M., T. Hirokawa, Y. Azuma, and J. Tajima. Color gamut of SOCS and its comparison to Pointer's gamut.

projection. DCI-P3 extends farther than AdobeRGB into the red region of the CIE 1931 chromaticity space. On the other hand, AdobeRGB extends farther into the green region.[3]

Rec. 2020 (for ITU-R Recommendation BT.2020) is a standard that has been promulgated for UHDTV (both 4K and 8K). It specifies many broadcast image characteristics including frame-rate, bit depth, resolution, and aspect ratio. It also specifies an RGB color space that is considerably larger than either AdobeRGB or DCI-P3. The Rec. 2020 RGB space is based on monochromatic primaries with wavelengths of 630 nm (red), 532 nm (green), and 467 nm (blue). No current displays can cover the entire Rec. 2020 gamut. The "2020" in the name is said to indicate that 2020 is the target year for implementation.[4]

2. Gamut Comparisons

The remainder of this paper shows graphical comparisons of the AdobeRGB, DCI-P3, Rec. 2020, and Pointer's gamuts, together with the MacAdam limits. I have chosen to use three different formats. These are (1) the familiar $x - y$ chromaticity diagrams based on the CIE 1931 *XYZ* color space; (2) diagrams in $a^* - b^*$ dimensions (*i.e.,* Lab color space); and (3) diagrams that plot chroma (C^*) as a function of lightness (L^*) for selected hue angles. The latter are based on the LCh transformation of the Lab color model. All the gamuts, as well as the MacAdam limits, vary with luminance. Therefore, for the first two graph types, gamuts are plotted for nine different values of L^*. The gamut boundaries represent the colors that are farthest from the white point, or neutral axis, for a given hue angle, *h,* and lightness, L^*. For these purposes, colors were grouped into 5º hue intervals, and lightness intervals of $\Delta L^* = 5$. A more complete description of the methods can be found after the Discussion.

Although $x - y$ chromaticity diagrams, usually with the spectral locus included, are commonly used for graphical representation of gamuts, they are far from ideal. For one thing, colors of different luminance are projected onto a single two-dimensional plane. That obscures the fact that gamut dimensions change with lightness; as will be evident in the following figures. Hence, one should make separate plots for various luminance levels. The second drawback is that chromaticity confounds hue and chroma (as defined in the LCh color model). For example, colors that have the same hue, but very different chromas can have identical chromaticity coordinates; and therefore "map" to exactly the same point in two-dimensional $x - y$ space. Even more problematic is that fact that that colors with the same $x - y$ chromaticity coordinates can have different hues and chromas.[5]

---

[3] To make things more confusing, there is a color space known as Adobe Wide Gamut RGB. It does not appear to have been widely used, and should not be confused with generic "wide-gamut" color spaces or displays.

[4] An excellent discussion of Rec. 2020 and other RGB color spaces, and their coverage of Pointer's gamut can be found in this article by Kid Jansen. Also included is an extensive summary of wide-gamut monitor panels.

[5] This latter assertion can be verified quite easily with Bruce Lindbloom's CIE Color Calculator. sRGB [255, 0, 0] and [1, 0, 0] both have D65 (*x, y*) coordinates (0.64, 0.33). Yet the chroma and hue of the first sRGB triplet are C^* =104.55 and h = 40.0º; whereas for the second triplet, they are C^* = 0.2773 and h = 19.4º.

Gamut plots in $a^* - b^*$ space still need to be made for different lightness levels. However, colors of the same hue but different chroma cannot fall on the same point in two-dimensional $a^* - b^*$ space: colors with greater chroma are farther from the origin. Furthermore, colors with the same $a^* - b^*$ coordinates will always have the same hue (h) and chroma (C^*). This difference between the xyY and Lab color models has an important consequence when separate gamut plots are made for different lightness levels. In $x - y$ chromaticity diagrams, gamut size appears maximal at the lowest luminance level. In Lab space, however, gamut size is maximal at intermediate lightness levels for most hues. That reflects the fact that maximum possible chroma decreases when colors become very dark or very light. That point is made clear by the plots of maximum chroma as a function of lightness (Fig. 4).

If there is a drawback to $a^* - b^*$ gamut representations it is that it is awkward, at best, to include the spectral locus. That is because the three-dimensional shape of the Lab color space is highly irregular; and many real colors will appear to be *outside* the spectral locus in two-dimensional $a^* - b^*$ space. However, so long as comparisons to the spectral locus are not required, I feel that $a^* - b^*$ plots are preferable to $x - y$ diagrams: they give a better representation of gamut shape and extent.

3. Results

Results are shown in Fig. 2 ($x - y$ chromaticity diagrams), Fig. 3 ($a^* - b^*$ diagrams), and Fig. 4 (C^* *vs.* L^* for selected hue angles). They will be easiest to summarize in list form.

3.1. The MacAdam limits lie close to the spectral locus and purple line at low lightness levels ($L^* \leq 40$). At higher lightness, the limits remain close to the spectral locus only in the yellow region of the spectrum (Fig. 2). In other words, theory tells us that for reflective colors, only yellows can have high luminance and high saturation simultaneously.

3.2. For most colors and lightness levels (L^*), the boundaries of Pointer's gamut are far from the MacAdam limits. In other words, the realized saturation of natural and man-made surface colors tends to be much less than the theoretical maximum. That is especially true for green, cyan, blue, and magenta colors at all lightness levels (Fig. 4). On the other hand, Pointer's gamut approaches MacAdam limits for red and yellow colors at low and intermediate lightness levels: the maximum saturation of real red colors approaches the theoretical maximum for $L^* \leq 40$.

3.3. The Rec. 2020 gamut exceeds or equals AdobeRGB and DCI-P3 at all lightness levels.

3.4. The Rec. 2020 gamut is also larger than Pointer's gamut everywhere.[6]

[6] At $L^* = 40$, coverage appears to be just less than 100%. At all other lightnesses shown, coverage exceeds 100%; often by considerable margins.

3.5. The AdobeRGB and DCI-P3 gamuts exceed Pointer's gamut in the green and blue regions (Figs. 3 and 4). In other color regions, coverage of Pointer's gamut depends on lightness.

3.6. In general, all three RGB gamuts are smaller than the MacAdam limits, although there are some excursions beyond the limits for colors close to the RGB primaries. For example, all three RGB spaces exceed the MacAdam limits in the red region (for some lightnesses, Fig. 4).

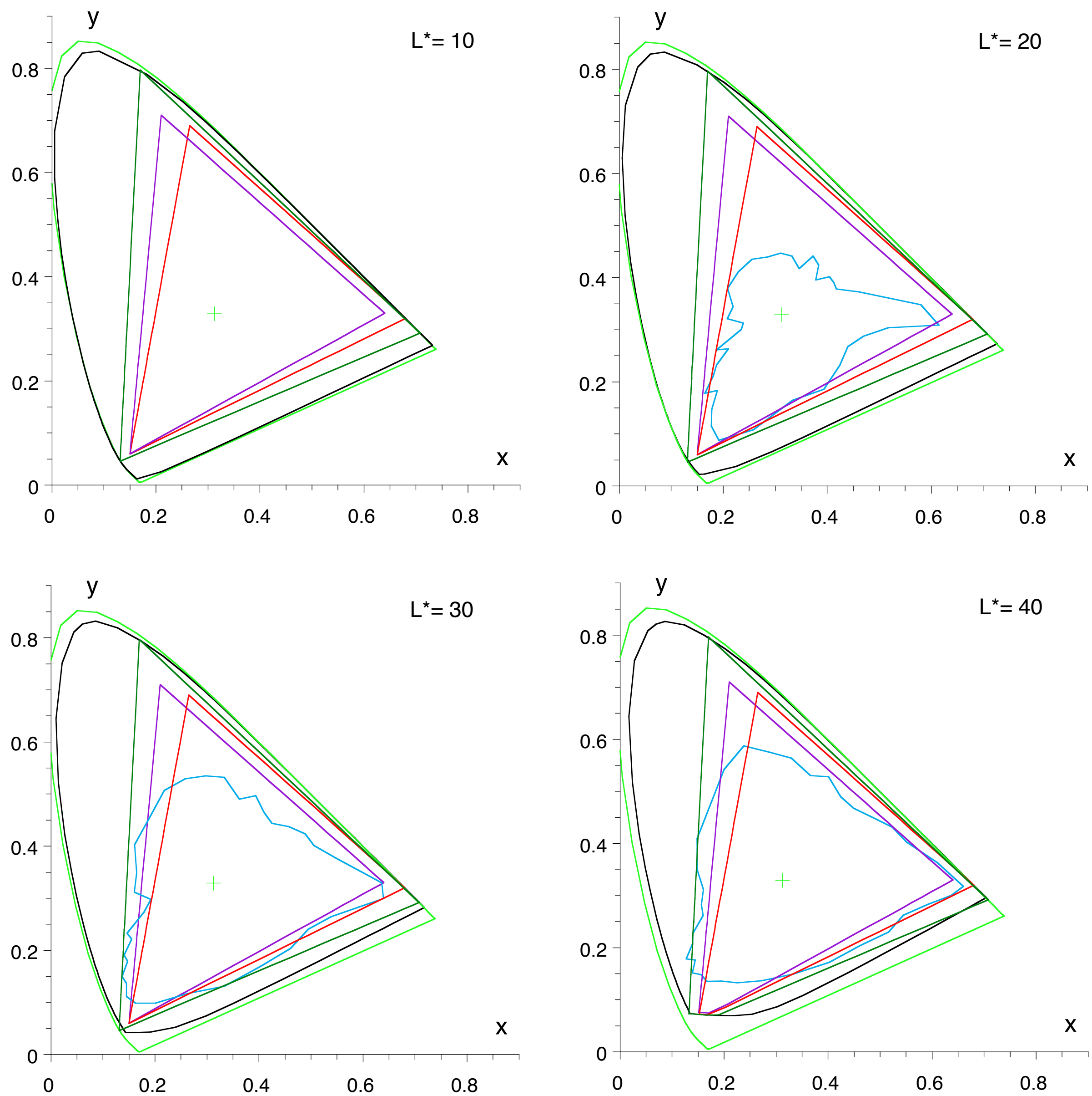


Fig. 2, Part 1. Gamuts in $x-y$ space. Black: MacAdam limits; dark green: Rec. 2020; purple: AdobeRGB; red: DCI-P3; blue: Pointer's gamut; light green: spectral locus and "purple line"; cyan cross: D65 white point. Pointer's gamut not published for $L^* = 10$.

3.7. At high lightness levels ($L^* \geq 80$), the Rec. 2020 gamut equals or exceeds the MacAdam limits in several color regions, notably green, cyan, blue, magenta, and red. The same is also true for AdobeRGB and DCI-P3, at least in the blue – magenta – red regions.

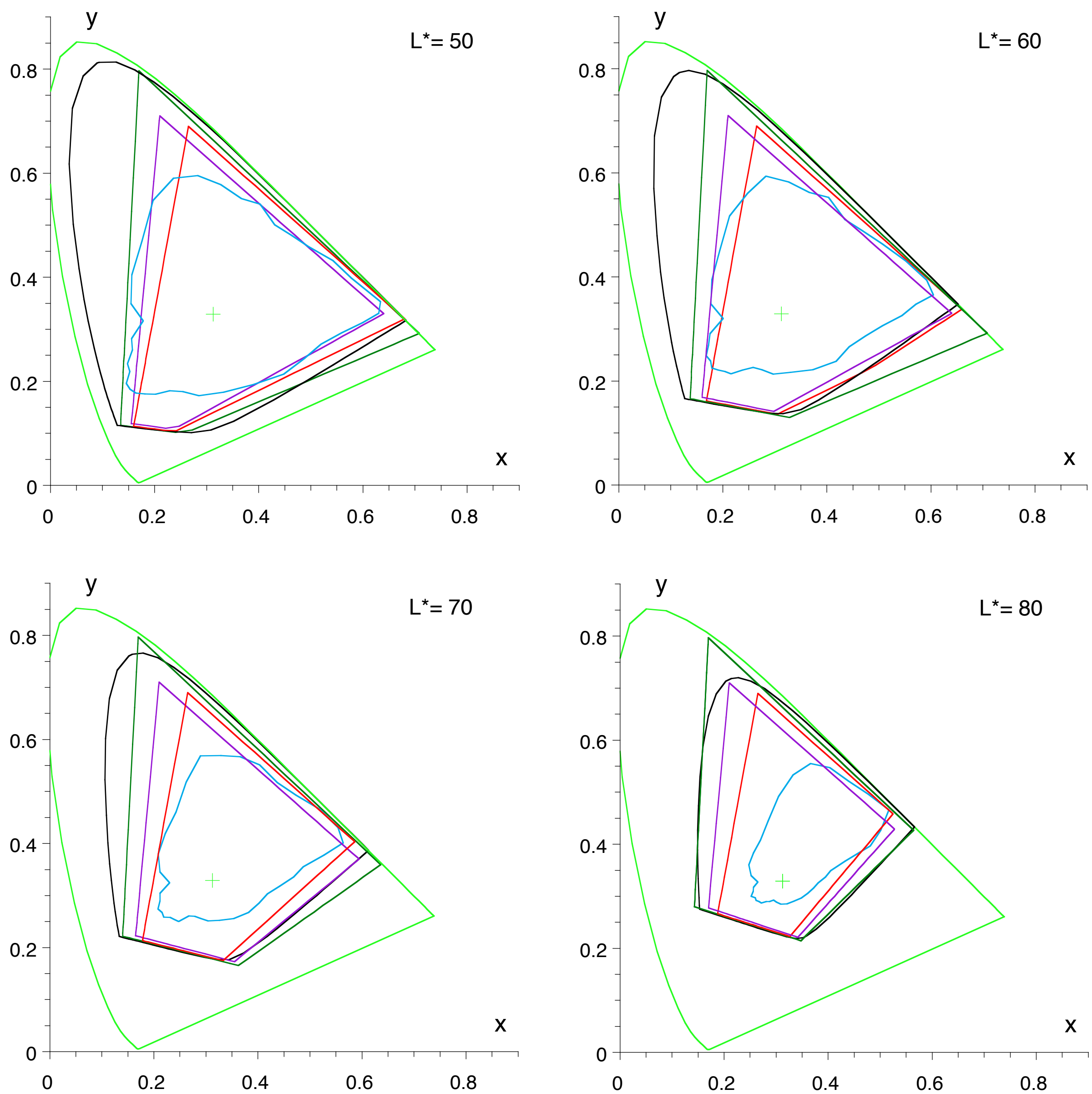


Fig. 2, Part 2. Gamuts in *x* – *y* space. Black: MacAdam limits; dark green: Rec. 2020; purple: AdobeRGB; red: DCI-P3; blue: Pointer's gamut; light green: spectral locus and "purple line"; cyan cross: D65 white point.

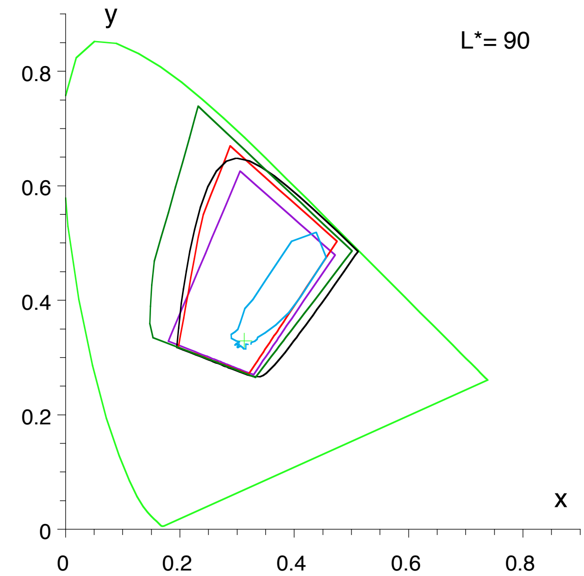
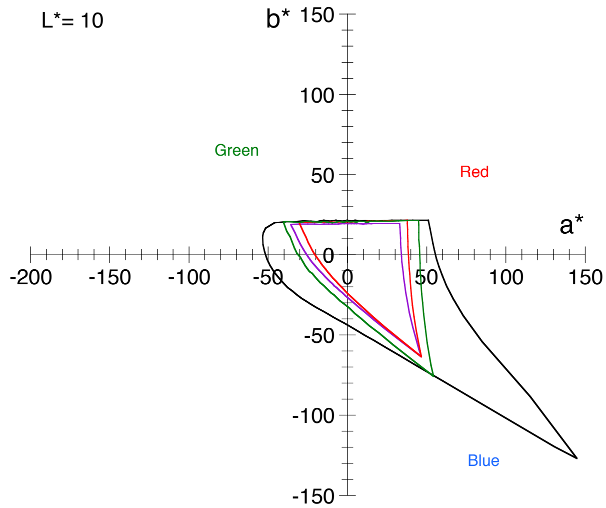
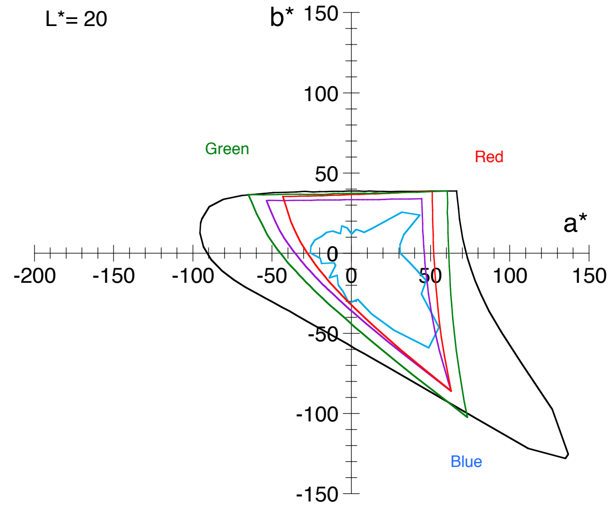
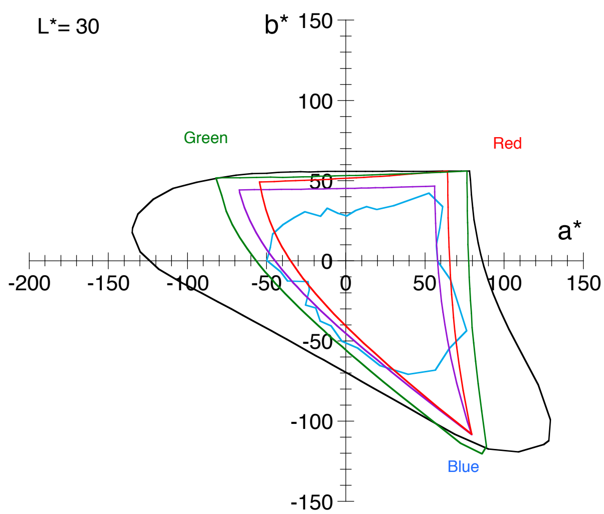
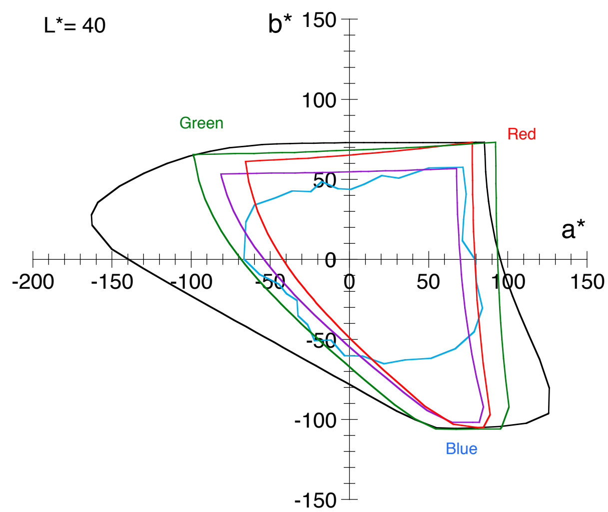


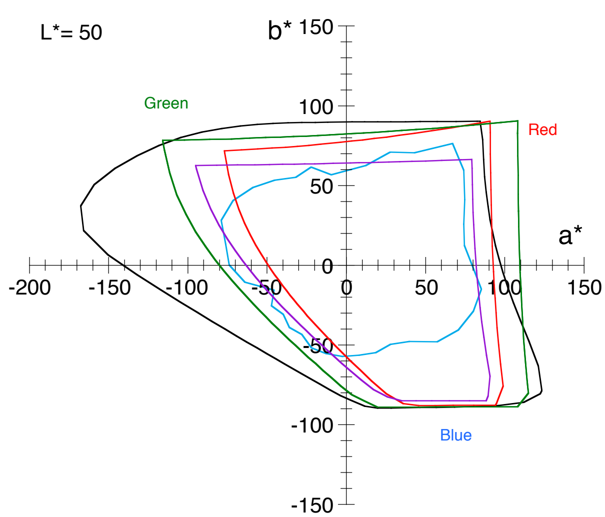
Fig. 2, Part 3. Gamuts in *x* – *y* space. Black: MacAdam limits; dark green: Rec. 2020; purple: AdobeRGB; red: DCI-P3; blue: Pointer's gamut; light green: spectral locus and "purple line"; cyan cross: D65 white point.

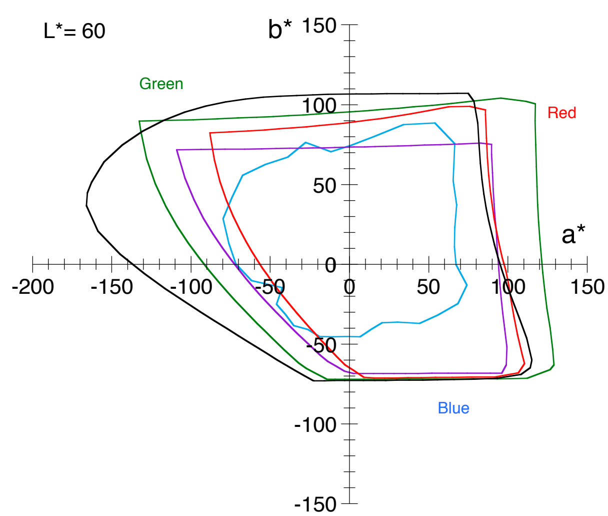

L*= 10
b*
a*
Green
Red
Blue
150
100
50
0
-50
-100
-150
-200
-150
-100
-50
0
50
100
150


L*= 20
b*
a*
Green
Red
Blue
150
100
50
0
-50
-100
-150
-200
-150
-100
-50
0
50
100
150


L*= 30
b*
a*
Green
Red
Blue
150
100
50
0
-50
-100
-150
-200
-150
-100
-50
0
50
100
150


L*= 40
b*
a*
Green
Red
Blue
150
100
50
0
-50
-100
-150
-200
-150
-100
-50
0
50
100
150


L*= 50
b*
a*
Green
Red
Blue
150
100
50
0
-50
-100
-150
-200
-150
-100
-50
0
50
100
150


L*= 60
b*
a*
Green
Red
Blue
150
100
50
0
-50
-100
-150
-200
-150
-100
-50
0
50
100
150

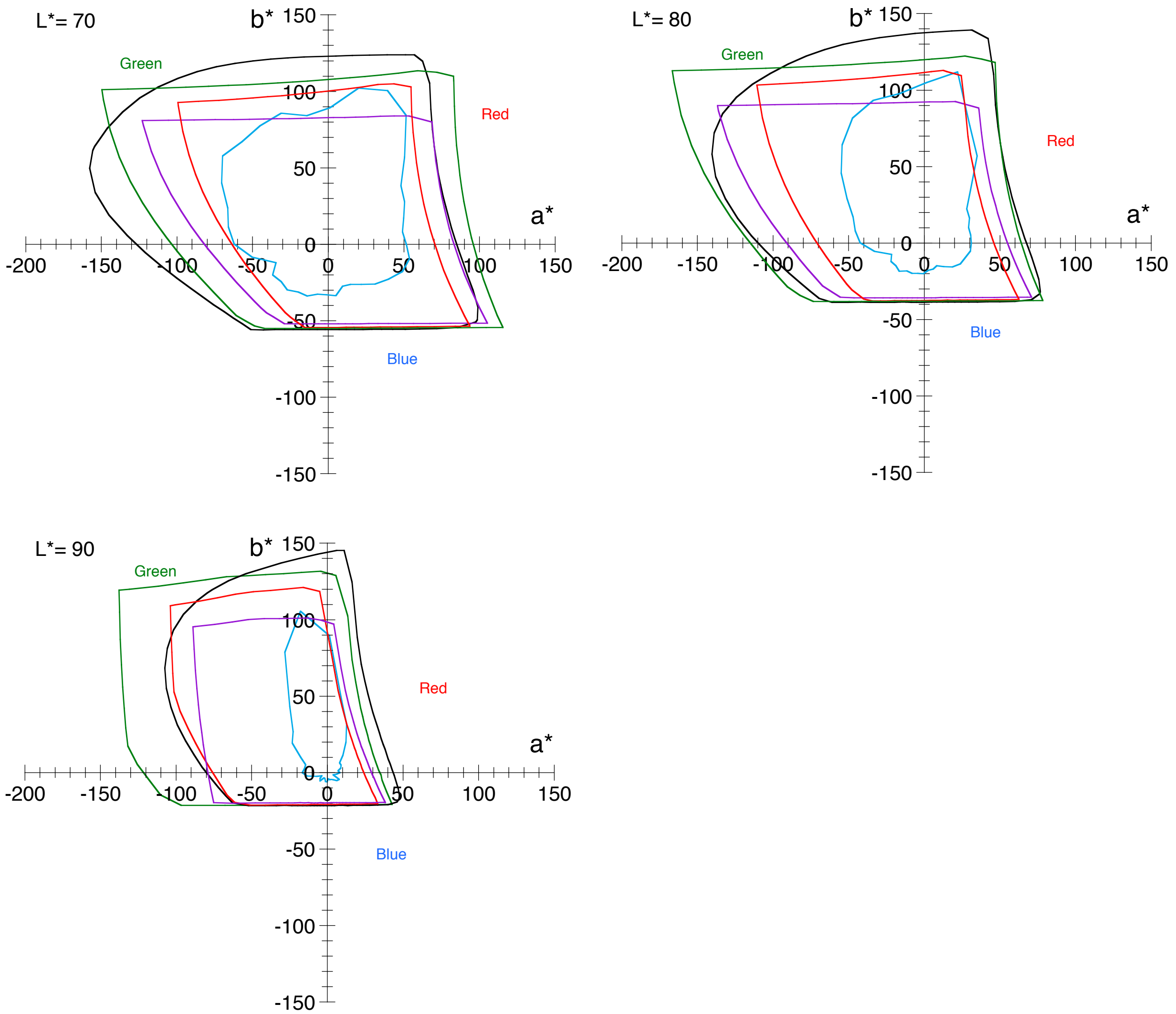


Fig. 3, Part 1 (previous page), Part 2 (this page). Gamuts in $a^* - b^*$ space. Black: MacAdam limits; dark green: Rec. 2020; purple: AdobeRGB; red: DCI-P3; blue: Pointer's gamut.

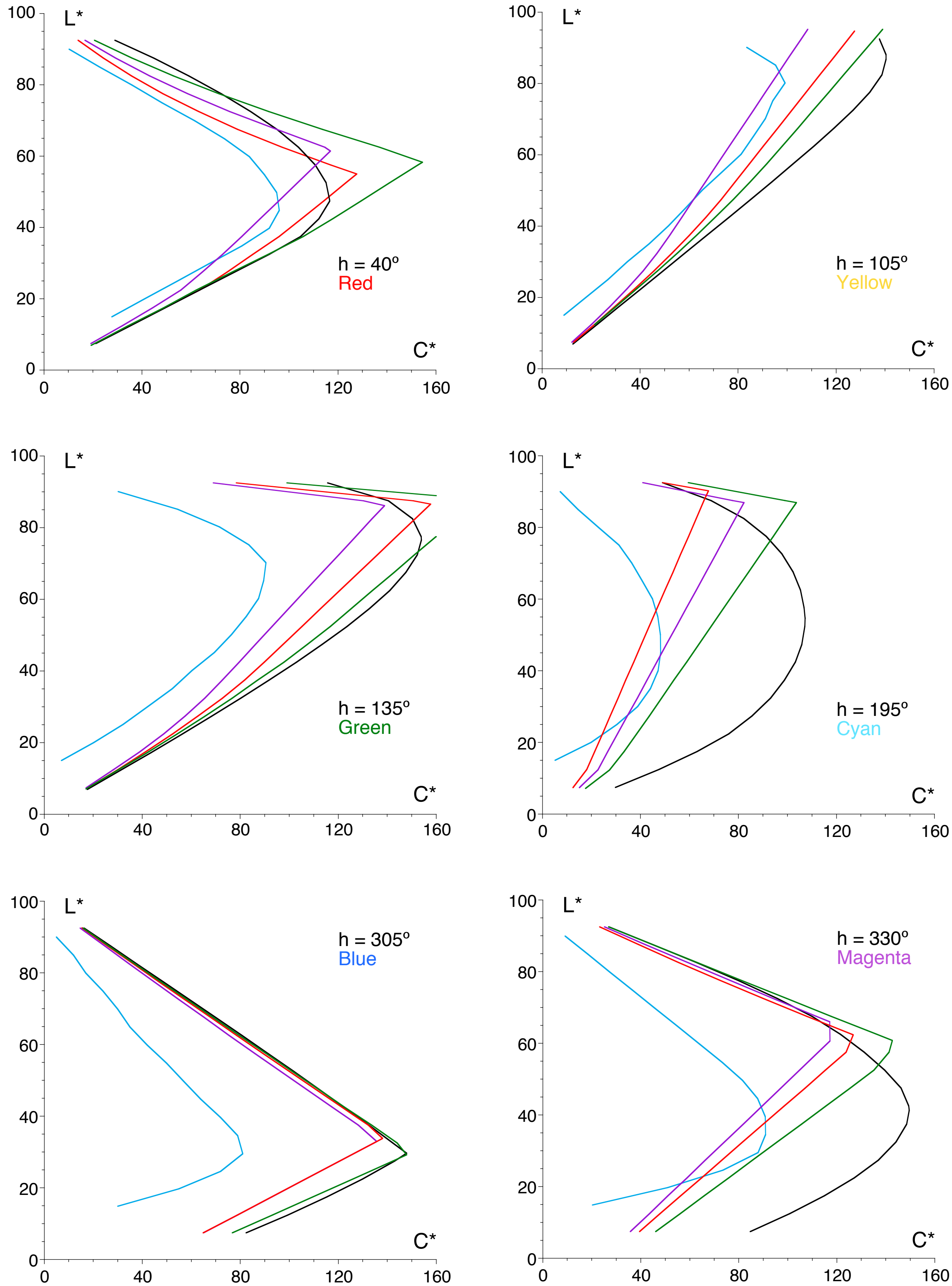


Fig. 4. Gamuts in *C** – *L** space for selected hue angles (*h*). The hue angles correspond approximately to sRGB red [255, 0, 0], yellow [255, 255, 0], green [0, 255, 0], cyan [0, 255, 255], blue [0, 0, 255], and magenta [255, 0, 255]. Black: MacAdam limits; dark green: Rec. 2020; purple: AdobeRGB; red: DCI-P3; blue: Pointer's gamut.

4. Discussion

If complete coverage of the gamut of real surface colors (*i.e.,* Pointer's gamut) is a goal, then the Rec. 2020 specification accomplishes that task. I do not know when, if ever, display panels that can reproduce all of the Rec. 2020 RGB space will be available. That will depend, I presume, on the ability to generate the Rec. 2020 primaries with sufficient brightness.

Coverage of the MacAdam limits is a different story. Rec. 2020 is the best of the three RGB spaces examined. However, at most lightness levels, the Rec. 2020 space is smaller than the region enclosed by the optimal color locus. The gap is especially large in the cyan region (for $L^* < 80$) and the magenta region (for $L^* < 50$) (Fig. 4). Only for $L^* \geq 80$ does the Rec. 2020 space approach complete coverage of the MacAdam limits (Figs. 2 and 3). This may be a moot point because the "missing" colors are outside Pointer's gamut, and remain theoretical, at least as surface colors.

That said, light sources can certainly produce colors that are outside Pointer's gamut, and outside the Rec. 2020 gamut. Furthermore, digital camera sensors are sensitive to all visible wavelengths; and can, in principle, capture any color visible to humans.[7] In order for displays to approach the saturation of optimal colors at lower luminance levels ($L^* \leq 60$) in all regions of the spectrum, or to approach the spectral locus in all regions, it will be necessary to use more than three "primary" colors. Such "multi-primary" displays are under consideration.[8] What is not clear, at least to me, is if there is much demand for such "super-wide-gamut" displays.

5. Methods

All three RGB color spaces use D65 for their reference white point. Therefore all results are computed for D65 illuminant. All figures were made with KaleidaGraph 4.5.2, by **Synergy Software**.

5.1. Pointer's Gamut

The boundaries of Pointer's gamut as functions of lightness (L^*) and hue angle (h_{ab}) are tabulated by Pointer (1980, his Table 2).[9] Pointer's data are for illuminant C. I converted his results to D65 using standard procedures.[10] Pointer grouped colors into lightness intervals of width 5, beginning with $L^* = 15 \pm 2.5$ and running through $L^* = 90 \pm 2.5$. Colors were grouped into hue angle intervals of 10º (*e.g.,* $h_{ab} = 30 \pm 5º$).

5.2. MacAdam Limits

MacAdam limits were determined using a C-language computer program written by me. The program inputs were the CIE 1931 color-matching functions and the D65 spectral power distribution. These were used to calculate all possible CIE *XYZ* trimstimulus values that obeyed the rules for optimal colors. That is, color-matching functions were either left unchanged or set to zero, thus simulating either

---

[7] Holm, J. Capture color analysis gamuts.

[8] Chan, C-C., et al. Development of multi-primary color LCD.
Brill, M. H., and J. Larimer. 2007. Metamerism and multi-primary displays.

[9] This table may be downloaded as an Excel spreadsheet from http://www.cis.rit.edu/research/mcsl2/online/cie.php

[10] Many useful formulas are available at www.brucelindbloom.com. This particular chromatic adaptation used the Bradford method.

complete reflectance or absorbance at each wavelength; and only two transitions between those states were permitted across the visible spectrum (Fig. 1). In order to determine the optimal color loci with sufficient smoothness, it was necessary to use 0.1 nm intervals for the color-matching functions and for the D65 spectral power distribution.[11] Computation of all possible optimal colors with 0.1 nm resolution takes less than ten seconds on a mid-2011 MacMini with 16GB RAM.

For each optimal color, *XYZ* coordinates were converted to *xyY, L*a*b*,* and *C*h* (ab) coordinates using standard formulas.[12] Colors were then assigned to lightness groups with width $\Delta L^* = 5$ (*e.g.,* $L^* = 10 \pm 2.5$, $L^* = 15 \pm 2.5$, etc.); and to hue angle groups of 5º (*e.g.,* $h_{ab} = 35 \pm 2.5$º).[13] The most chromatic color in each $L^* - h_{ab}$ group was then used to draw the locus of optimal colors for that L^* value. "Most chromatic color" was defined in two ways. For figures in $a^* - b^*$ space (Fig. 3) and its transformed $C^* - L^*$ space (Fig. 4), I chose the color with the greatest value of C^* in each group: that is, the color farthest from the origin, or neutral axis. For figures in $x - y$ chromaticity space (Fig. 2), I chose the color that lay at the greatest Euclidean distance from the D65 white-point chromaticity coordinates. For all intents and purposes, these two definitions appear to be equivalent.

5.3. RGB Gamuts

Gamut boundaries for each of the RGB color spaces were obtained by examination of all possible *RGB* triplets. In order to obtain smooth boundaries, it was necessary to use 10-bit precision. That is, *R, G,* and *B* were allowed to assume values from $0 - 2^{10} - 1 = 1023$. Thus, for each color space, more than one billion triplets were tested. Each triplet was converted to CIE 1931 *XYZ* coordinates and then to *xyY, L*a*b** and *C*h* (ab) coordinates using the formulas and methods available at www.brucelindbloom.com. All calculations were done with a C-language program written by me. For each RGB space, the calculations required almost eight minutes on the MacMini. I used the AdobeRGB-to-*XYZ* conversion matrix provided on the Lindbloom website. I found no "official" sources for the DCI-P3-to-*XYZ* and Rec. 2020-to-*XYZ* conversion matrices. Therefore, I derived those using the procedures described by Lindbloom. The matrices are provided in the Appendix. I did find one other DCI-P3-to-*XYZ* conversion matrix.[14] It is very similar to mine, which suggests that both are at least approximately correct. I assumed a gamma of 2.2 for AdobeRGB and 2.6 for DCI-P3. I could not find any reference to a gamma for Rec. 2020. Therefore, the Rec. 2020 gamut boundaries were calculated on the assumption of no gamma. The color resulting from each *RGB* triplet was assigned to a $L^* - h_{ab}$ grouping, and the most chromatic color in each group was selected following the same procedures just described for optimal colors.

---

[11] A downloadable spreadsheet with 1.0 nm intervals for the color matching functions and the D65 SPD can be obtained from http://www.cis.rit.edu/research/mcsl2/online/cie.php. I used simple linear interpolation to estimate 0.1 nm resolution.

[12] www.brucelindbloom.com

[13] Note that these hue angle groups are one-half the width of those tabulated by Pointer (1980).

[14] https://github.com/w3c/csswg-drafts/commit/9b41605f56722aad70c48c39f25f3f335f38008e

REFERENCES

MacAdam, D.L. 1935a. The theory of the maximum visual efficiency of colored materials. J. Optical Society of America. 25:249-252.

MacAdam, D.L. 1935b. Maximum visual efficiency of colored materials. J. Optical Society of America. 25:361-367.

Pointer, M. 1980. The gamut of real surface colours. Color Research and Application. 5:145-155.

RESOURCES

Jansen, Kid. 2014. The Pointer's Gamut: The coverage of real surface colors by RGB color spaces and wide gamut displays. http://www.tftcentral.co.uk/articles/pointers_gamut.htm
A comparison of many RGB color spaces to Pointer’s gamut. Includes some discussion about the feasibility of making displays that can reproduce particular RGB spaces, and summarizes the many wide-gamut displays available at the time of writing (February 2014)

Lindbloom, Bruce. www.brucelindbloom.com. An excellent on-line color calculator. Also an excellent source for the many matrices and mathematical formulas used for transforming among color spaces and color models, calculating color differences, etc.

Many useful data sets can be found at the Munsell Color Science Laboratory, Rochester Institute of Technology. http://www.cis.rit.edu/research/mcsl2/online/cie.php.

Appendix

A.1. DCI-P3-to-XYZ Conversion Matrix (computed by the author)

Prior to using this matrix, *RGB* triplets must be linearized (inverse companded).

[0.4866327	0.2656632	0.1981742]
[0.2290036	0.6917267	0.0792697]
[0.0000000	0.0451126	1.0437174]

A.2. DCI-P3-to-XYZ Conversion Matrix (obtained from https://github.com/w3c/csswg-drafts/commit/9b41605f56722aad70c48c39f25f3f335f38008e)

[0.4865709486482162	0.26566769316909306	0.1982172852343625]
[0.2289745640697488	0.6917385218365064	0.079286914093745]
[0.0000000000000000	0.04511338185890264	1.043944368900976]

Note that the first eight elements of the above two matrices are identical when rounded to four decimal places.

A.3. Rec. 2020-to-XYZ Conversion Matrix (computed by the author)

[0.6370102	0.1446150	0.1688448]
[0.2627217	0.6779893	0.0592890]
[0.0000000	0.0280723	1.0607577]