

Limits of Resolution. 2. Diffraction

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Summary

The effect of diffraction blur on resolution of black and white line-pair images is modeled by aggregation of a large number of blur circles. The contrast ratio of the line-pairs declines to 50% when the blur circle diameter is about 79% of the width of a line-pair. Contrast falls to about 7% when the blur circle diameter reaches 115% of the line-pair width. These results imply aperture-specific diffraction-limited resolutions. For example, with 50% contrast, the theoretical limit of image resolution is approximately 209 line-pairs per millimeter (lp/mm) at f/4. No other sources of blur are considered. Results are compared to calculated aperture-specific resolution limits published elsewhere. Resolutions obtained here by simulation are *less* than those calculated from "standard" formulas when line-pair contrast is 50% or less. However, in the case of high contrast (80%), simulated resolution limits are *greater* than values obtained by formulas. In order for cameras with "full-frame", APS-C, and m4/3 sensors to achieve resolutions closer to the theoretical diffraction limits, it will be necessary to increase the number of sensor photosites several-fold. It is not clear how much, if any, improvement will be required in lenses.

Key words: diffraction blur, Airy disk, diffraction-limited resolution, sensor-limited resolution, line-pairs, Zeiss Otus 55mm f/1.4, Nikon D800E, Clarkvision

1. Introduction

Given a perfect lens, and no other sources of blur, an infinitesimal point in the object field will be focused to an infinitesimal point in the image. We can't see infinitesimal points: they are too small and vanishingly dim. The identifiable objects that we can see are composed of aggregations of many (effectively an infinite number) of infinitesimal points. If we add blur to the picture, the infinitesimal object field points are focused as fuzzy circles. The fuzzy circles can have more or less well-defined dimensions, say 18.3 µm diameter. In principle, an isolated blur circle could be seen with a magnifier, although it would also be vanishingly dim. We never seen isolated blur circles in an image because each one overlaps with a very large number of other blur circles that correspond to nearby points in the object field. The effect of blur is to make the edges of image elements fuzzy. At low levels, blur reduces detail contrast. At higher levels, image details can be lost and resolution decreases.

The previous paper in this series considered the effect of sampling frequency (photosite size) on image resolution in the absence of blur.[1] In this paper, I will develop a model of the effect of blur on image resolution. Although the model is general with respect to the source of

[1] Service, Phil. 2014. Limits of Resolution. 1. Sampling Frequency

blur, the present discussion refers specifically to diffraction blur, which is a function of aperture and is unavoidable in photography. This paper *will not* consider the interaction between diffraction blur and photosite size. However, I will consider the interaction between blur and overall sensor size with regard to resolution. In this context, sensor size is just a proxy for the physical dimensions of the image produced by the lens.

1.1. Terminology

In order to avoid confusion, we need to define a few terms. The *lens image* is the image that is formed by the lens. The lens image is an effectively continuous, analog representation of the external world — *object field* — in front of the lens. In this paper, I am concerned with the effect of blur on the lens image. *Resolution* in the present context means line-pairs per millimeter (lp/mm). In general, a *contrast ratio* will be associated with a resolution measure. The contrast ratio for line-pairs is the maximum difference in lightness values of adjacent light and dark lines, divided by their sum. A *perfect lens* is a lens with no optical aberrations. In the absence of diffraction, it would produce a image with no blur. A perfect lens is assumed in everything that follows.

1.2. Diffraction Blur

What is the proper calculation for the size of the diffraction blur circle? The standard procedure is to use the diameter of the central bright circle of the Airy diffraction pattern circumscribed by the middle of the first dark ring — the area usually referred to as the Airy disk.[2] Taking that as the size of diffraction blur seems to be purely arbitrary, perhaps justified by the fact that there is a convenient formula. In photographic applications, the diameter, c_a, of the Airy disk is

$$c_a = \frac{2.44\lambda N}{k} \qquad (1)$$

where N is aperture (f-number), λ is the wavelength of light (generally taken to be about 550 nm), and k is a scaling factor. If nanometers are the units of wavelength and it is desired to express diffraction blur in micrometers (microns), then k = 1,000.

The difficulty with the conventional formula is that it ignores the fact that the intensity of the Airy disk declines markedly well in advance of the first minimum of the diffraction pattern. Elsewhere, I have attempted to equate diffraction blur to defocus blur.[3] The latter is estimated using a geometric argument based on ray tracing, and assumes that intensity is uniform across the defocus blur circle. My tests suggest that for photographic purposes, the equivalent amount

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[2] The Airy diffraction pattern is produced when light is shone through a small circular aperture. A lens is not required. The pattern consists of a central bright disk surrounded by progressively dimmer rings separated by dark minima. See Appendix 1 for an illustration.

[3] Service, Phil. 2014. Optimal Aperture in Photography. 2. Testing the Theory — Blur Equivalence
Service, Phil. 2014. Optimal Aperture in Photography. 3.Testing the Theory — The Diffraction Blur Coefficient

of blur is obtained when the diffraction blur diameter is about 70% of the value given by Eq. 1. In fact, at 70% radius, the intensity of the Airy disk has declined to only about 11% of its maximal central value; and about 78% of the total power in the diffraction pattern is contained within the corresponding region. See Appendix 1 for more details. In much of the analysis that follows it will not be necessary to use the absolute size of the diffraction blur circle. However, when it is, I will use the 70% "correction".[4] Thus,

$$c_a = \frac{1.71\lambda N}{k} \qquad 2$$

Note that this correction means that a one stop smaller aperture (larger numerical f-number) is required to produce a given amount of diffraction blur.

2. The Model

2.1. Lens Image Line-pairs in the Absence of Blur

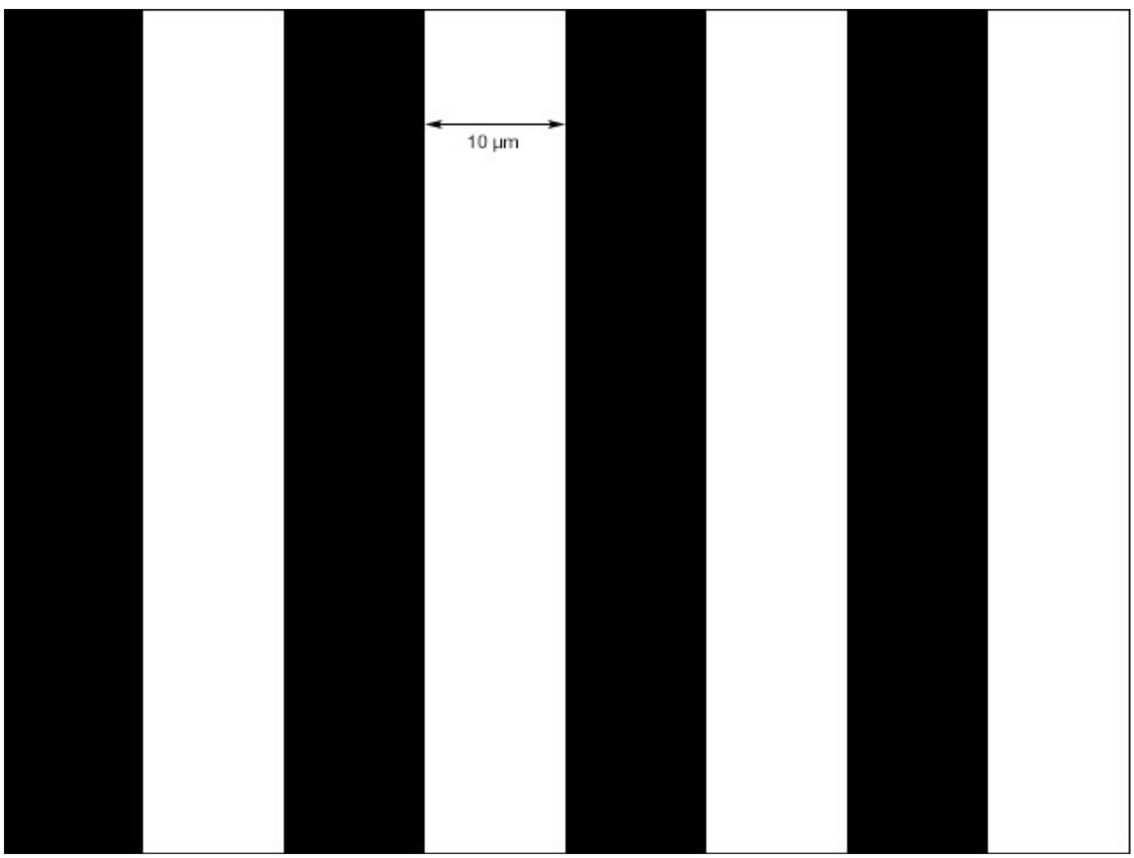

Fig. 1. Idealized lens image of line-pairs rendered without blur and 100% contrast ratio.

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[4] I will also assume $\lambda = 550$ nm, and $k = 1{,}000$, so that the units of the blur circle will be microns.

Fig. 1 depicts a portion of an idealized lens image of line-pairs without blur. The black lines are assumed to receive no light and have a lightness value = 0. The white lines have a lightness value = 1. Thus, the contrast ratio of this idealized image is (1 - 0) / (1 + 0) = 1. The actual width of the black and white lines is not important at the moment. However, for the sake of concreteness, I have indicated that the lines in the image are 10 µm wide. Equivalently, there are 50 lp/mm. The goal of this paper is to describe the effect of diffraction blur on this image of line pairs.

Fig. 2 is a graphical representation of the line pairs in Fig. 1. In this case lightness on the vertical axis is plotted against distance on the horizontal axis. Graphical descriptions of line pairs are much easier to produce than pictorial ones, especially once blur is added to the picture.

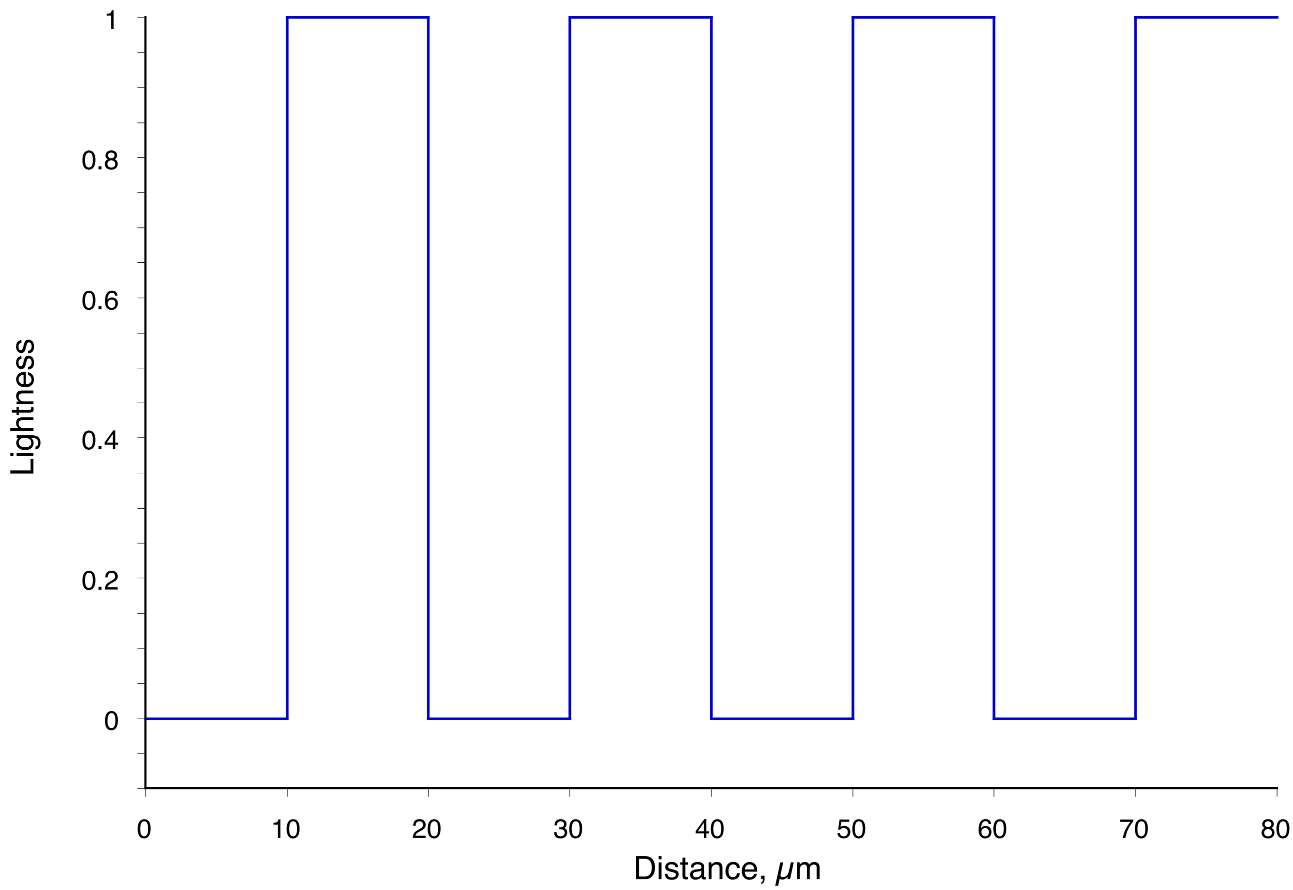

Fig. 2. Graphical representation of the idealized line-pairs in Fig. 1. No blur, 100 % contrast.

2.2. Blur

I start by assuming that scattering of light due to blur is a one-way phenomenon. That is, light is scattered from the white lines in the line-pair image to the black lines (which receive no light in the absence of blur). The net result is that the black lines become less black, and the white lines become less white (because they receive less light as a result of scattering), and the borders between white and black lines become fuzzy. In principle, we need to model the aggregate effect of an infinite number of vanishingly dim blur circles whose centers lie within the white lines of the line-pair image. That sounds like an application for calculus. However, my approach will be to model a large, but finite, number of blur circles and to sum their effects.

Imagine that each white line in the image of line-pairs is divided into a very large number of narrow horizontal "strips". Each horizontal strip within a single white line is composed of 1,000 "points". Each of those points will serve as the center of a blur circle. Each point, and therefore each blur circle, accounts for 1/1000 of the light that is contained in that horizontal

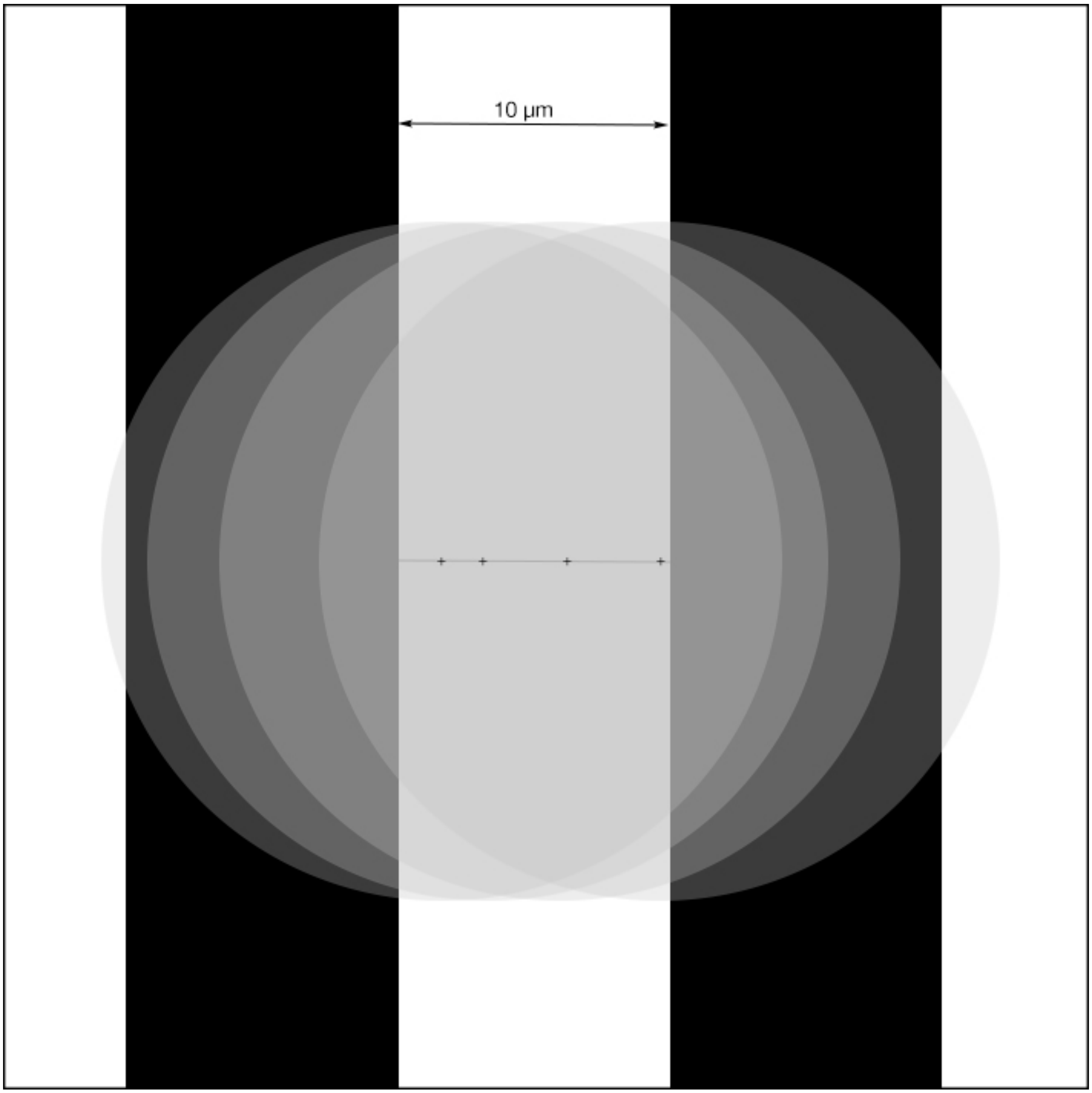

Fig. 3. Four of the 1,000 blur circles that would be centered along a narrow horizontal strip in a white line of line-pair image. Blur circle diameter is 25 μm.

strip (in the absence of blur). The model is illustrated in Fig. 3, showing just four of the thousand blur circles that would be located along that particular strip. These blur circles are scaled to have a diameter of 25 μm. Several points are illustrated in Fig. 3. First, if blur circles are large enough, they can overlap black lines on both sides of white line. Second, large blur circles may even overlap a white line beyond a neighboring black line. Third, if blur circles have diameters less than the width of a white line, then, depending on the location of their centers, they may not overlap and scatter light to a neighboring black line. Fourth, it is important to note that the amount of light scattered by any blur circle *into* a black line decreases with distance from the edge of the black line. This is perhaps easiest to visualize in Fig. 4, where only one blur

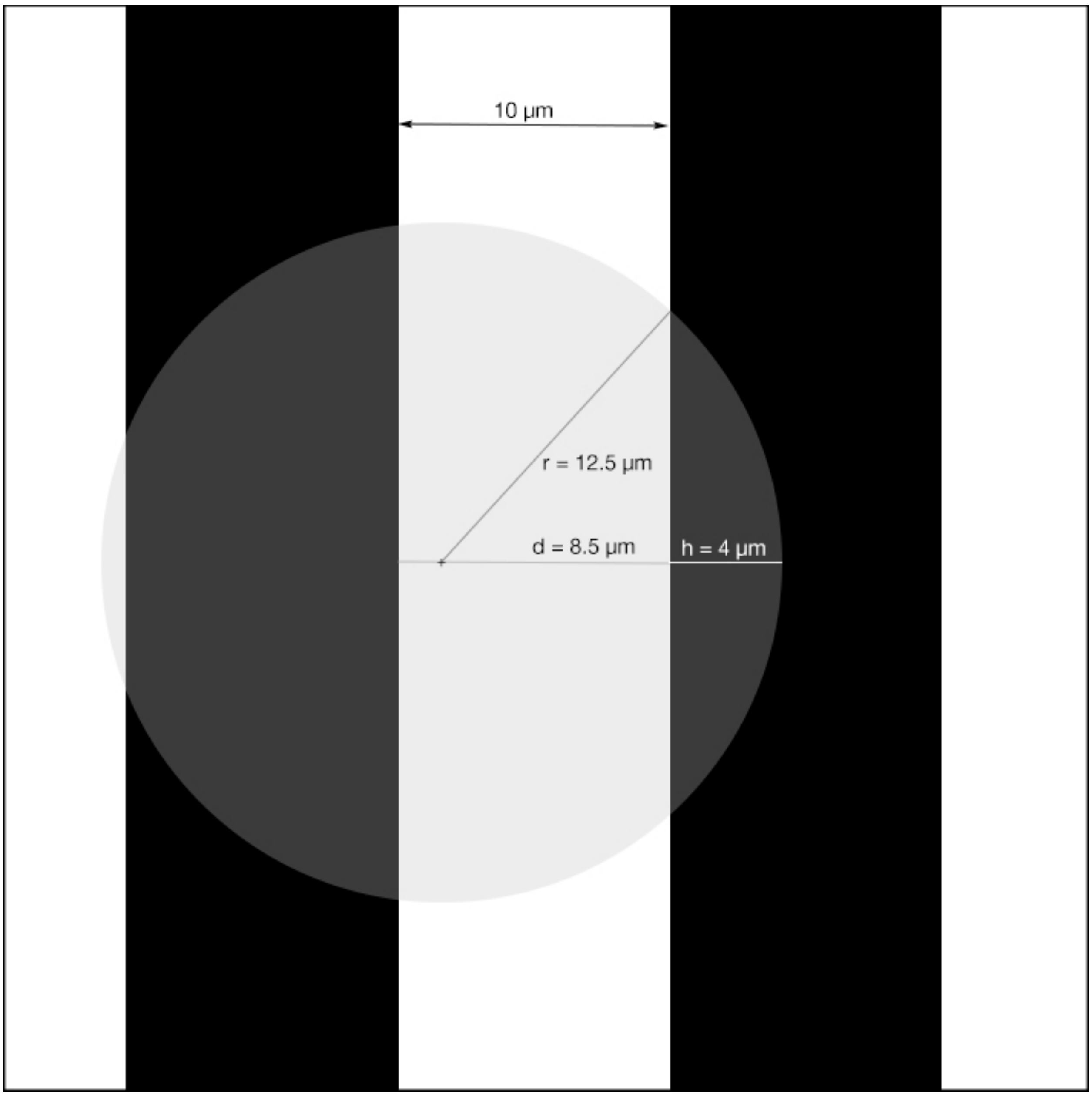

Fig. 4. A single blur circle originating from a white line, depicting segments of overlap with two adjacent black lines.

circle is shown. Consider the area of overlap of the blur circle with the black line on the right. Imagine that the overlap area is subdivided into a number of vertical strips. The strips become shorter, and thus have less area, as one moves from left to right across the area of overlap. Less area at progressively greater distances means less light transferred to progressively greater distances into the black line.

2.3. Calculation of Light Scattering by Blur

The gory details of the calculations are left to Appendix 2. The essence is that light scattering is modeled by the area of the blur circle that overlaps a black line.[5] That area is a circular segment — the 4 μm overlap on the right side of the blur circle in Fig. 4 — or a portion of a circular segment — the left side of the blur circle in Fig. 4. The calculation is straightforward. For the example illustrated in Fig. 4, the total area of the blur circle is 491 μm^2. The area of the segment that overlaps the black line to the right is 51 μm^2. The area of the blur circle that overlaps the black line on the left is 201 μm^2.[6] Thus, the total area of overlap on black lines is 252 μm^2, or about 51% of the area of the blur circle. That represents the amount of light attributable to that point in the white line that is lost to the black lines because of blur. A much larger set of essentially similar calculations is then required to distribute that lost light within each of the black lines. The process is then repeated for each of the other 999 blur circles located along the horizontal strip within that white line. Then, the total light gained at each horizontal position in the black lines is obtained by summing the results for each of the 1,000 blur circles. It is necessary to do this calculation for only one horizontal strip in a single white line. In practice, dimensionless units are used. The only variable then becomes the radius (or diameter) of the blur circle *relative* to the width of a line (or line-pair). For ease of computation, I constrained the diameter of the blur circle to be ≤ 2 line-pairs.

I wrote a short OS X command line tool in C in order to perform the calculations. The output consisted of the light intensity at each of 4,000 locations across two line-pairs. Results were visualized as graphs, similar to Fig. 2, that were created with Kaleidagraph.

2.4. Preliminary Conclusions

It is possible to make some preliminary conclusions without computation. For example, in order for the middle of a black line to remain maximally black — that is, receive no scattered light — and for the center of a white line to remain maximally white, the radius of the blur circle must be less than half the width of a line. Equivalently, in order for the lens image to retain

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5 The blur circle is assumed to have uniform intensity, unlike an Airy disk.

6 This calculation requires computing the area of the entire segment defined by the near edge of the black line to the left of the blur circle center. That segment includes the part of the blur circle that overlaps the white line on the far left. Then the area of that small segment defined by overlap on the leftmost white line is subtracted from the area of the larger segment. The light lost to the leftmost white line does not need to be subtracted from the white line that contains the blur circle center. Reciprocal exchange of light will occur between any pair of “points” in white lines for which there is overlap of blur circles.

100% contrast,[7] the diameter of the blur circle must be less than the width of a line, or less than half the width of a line pair. To be concrete, if we wish to produce a lens image with 50 lp/mm resolution and 100% contrast, the diffraction blur circle diameter must be less than 10 µm. By Eq. 2, the smallest aperture that would fulfill that criterion would be f/10 (rounded to nearest larger 1/3 stop).[8] If our criterion were more stringent, such that the central third or half of each line show no effects of blurring, the minimum aperture would be correspondingly smaller: f/7.1 or f/5 in the case of a lens image with 50 lp/mm. Note that these represent "best case scenarios"

Table 1. Relative Blur Circle Diameter and Line-pair Contrast

Blur Circle Diameter (% of line-pair width)	Maximum Contrast Ratio (%)	Proportional Part of Line-pairs Producing 100% Contrast (% of line-pair width)
10	100	80
20	100	60
30	100	40
40	100	20
50	100	–
60	84	–
70	65	–
80	48	–
90	34	–
100	22	–
110	12	–
115	7	–
120	3*	–
150	17**	–

* Repeating pattern of two local lightness maxima per black line and one global maximum per white line, together with one global minimum per black line and two local minima per white line.

** Inverted lightness pattern, with maximum in "black" line and minimum in "white" line.

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7 The contrast, or contrast ratio, is defined as the maximum difference in lightness between neighboring light and dark lines, divided by the sum of their lightnesses.

8 If we use Eq. 1, instead, the smallest aperture becomes f/7.1

— sampling of the lens image by the camera sensor would most likely degrade maximum contrast in the image produced by the sensor.

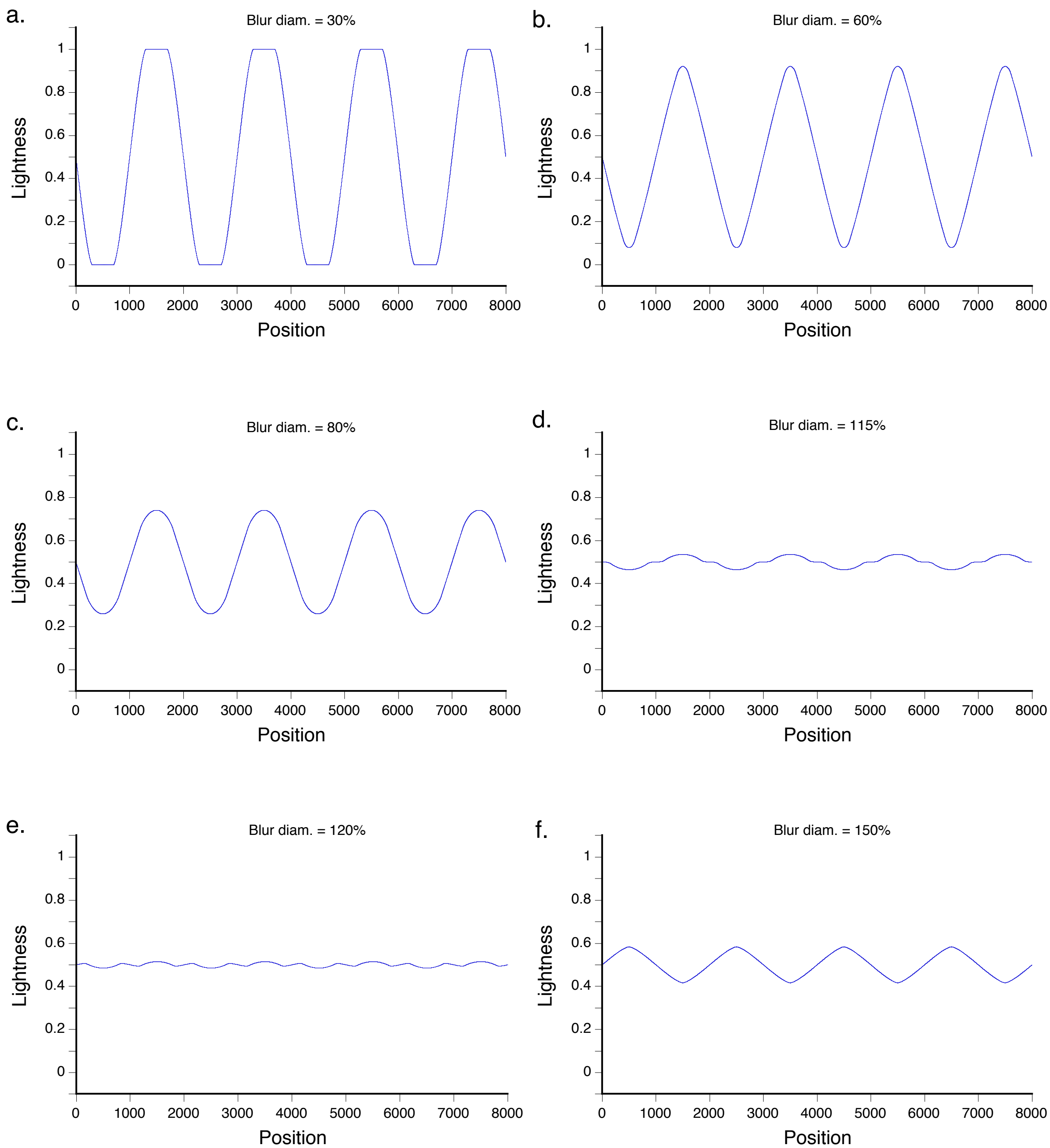

Fig. 5. Lightness as a function of horizontal position and relative blur circle diameter for four line-pairs. Lightness varies from 0 (black) to 1 (white). Position is in dimensionless units. Black lines in the unblurred image occupy positions 0 —999, 2000 – 2999, 4000 – 4999, and 6000 – 6999. White lines occupy positions 1000 – 1999, 3000 – 3999, 5000 – 5999, and 7000 – 7999.

3. Results

3.1. Blur Size and Contrast

The effect of relative blur diameter on line-pair contrast is summarized in Table 1. Several points can be made. (1) If the blur diameter is less than about 40% of the width of a line-pair, the lens image retains 100% contrast, and the completely black and completely white regions comprise a relatively large proportion of each line-pair (Fig. 3a). (2) The lens image has 50% contrast when the blur diameter is slightly less than 80% of the line-pair width (Fig. 3c). (3) Contrast decreases in an orderly fashion with increasing blur diameter until the blur circle to line-pair ratio is about 115%, at which point contrast has fallen to about 7% (Fig. 3d).[9] (4) Further increases in blur diameter produce a pattern with several local maxima and minima in the blur pattern (Fig. 3e). (5) When the ratio of blur circle diameter to line-pair width is 150%, the contrast returns to a pattern of one maximum and one minimum per line pair. However, the pattern is inverted — the maximum occurs where a line should be black and the minimum where a line should be white (Fig. 3f).

Fig. 6 is a simulated depiction of blurred line pairs with a 50% contrast ratio, which occurs when the blur circle diameter is about 78.75% of the width of a line-pair (see Fig. 5c).

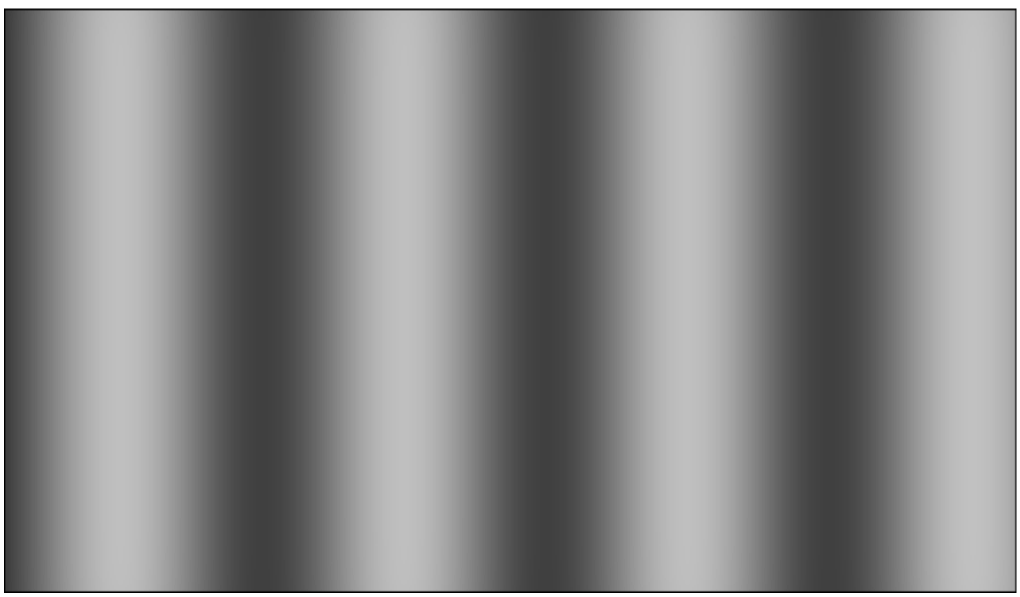

Fig. 6. Simulation of blurred line-pairs with 50% contrast. Made by applying the Photoshop Gaussian Blur filter to Fig. 1. See Fig. 5c for a graphical representation of this figure.

[9] By "orderly fashion" I mean that there is one lightness maximum and one minimum per line-pair, and that the maximum occurs in a "white" line and the minimum in a "black" line.

**Table 2. Theoretical Resolution Limits (lp/mm) as a Function of Aperture and Contrast Ratio
"Corrected" Estimate of Diffraction Blur***

Aperture	Diffraction Blur Dianmeter (µm)	Contrast Ratio						
		20%	50%	80%	100/20%	100/40%	100/60%	100/80%
1.4	1.32	772	598	471	304	228	152	76
2	1.88	540	419	330	213	159	106	53
2.8	2.63	386	299	236	152	114	76	38
4	3.76	270	209	165	106	80	53	27
5.6	5.27	193	150	118	76	57	38	19
8	7.52	135	105	82	53	40	27	13
11	10.35	98	76	60	39	29	19	10
16	15.05	68	52	41	27	20	13	7
22	20.69	49	38	30	19	14	10	5

* Diffraction blur diameter calculated by Eq. 2.

**Table 3. Theoretical Resolution Limits (lp/mm) as a Function of Aperture and Contrast Ratio
Conventional Estimate of Diffraction Blur***

Aperture	Diffraction Blur Dianmeter (µm)	Contrast Ratio						
		20%	50%	80%	100/20%	100/40%	100/60%	100/80%
1.4	1.88	541	419	330	213	160	106	53
2	2.68	379	293	231	149	112	75	37
2.8	3.76	271	210	165	106	80	53	27
4	5.37	189	147	116	75	56	37	19
5.6	7.52	135	105	83	53	40	27	13
8	10.74	95	73	58	37	28	19	9
11	14.76	69	53	42	27	20	14	7
16	21.47	47	37	29	19	14	9	5
22	29.52	34	27	21	14	10	7	3

* Diffraction blur diameter calculated by Eq. 1.

3.2. Diffraction-Limited Theoretical Resolution for a Perfect Lens

Tables 2 and 3 give the theoretical resolution limits (lp/mm) for a perfect lens as a function of aperture and contrast. Table 2 assumes that diffraction is computed according to Eq. 2, and Table 3 uses the common formula for the Airy disk diameter — Eq. 1. Note that these

results are independent of sensor size, photosite size, and focal length. These theoretical resolutions are higher than generally obtained in tests of lenses on APS-C and "full-frame" sensors. For example, the peak MTF50 center resolution reported for the Zeiss Otus 55mm f/1.4 on a Nikon D800E was 55.6 lp/mm at f/5.6.[10] Depending on how diffraction blur is calculated, this is only about one-third to one-half of the theoretical limit at f/5.6 — for what is generally acknowledged to be one of the very sharpest lenses available for normal photography.

4. Sensor Size and Resolution

Imagine for the sake of illustration that we have made a poster with 1,800 black and white line-pairs. Our task is to take frame-filling pictures of the entire poster with"full-frame" and m4/3 cameras — cameras whose sensors are, very approximately, 36mm and 18mm, respectively, on their long dimensions. The lens images will contain 50 lp/mm for the full-frame camera, and 100 lp/mm for the m4/3 camera, which translate into line-pair widths of 20 and 10 µm, respectively. We use both cameras at f/8, which by Eq. 2 means that the diffraction blur diameter is 7.52 µm. In relative terms, the blur in the full-frame image is only 37.6% of the line-pair width; while it is 75.2% of the line-pair width in the m4/3 image. Assuming perfect lenses, the full-frame image will have 100% contrast, while the m4/3 image will have perhaps 55% contrast (Table 1). Thus, all other things being equal, including aperture and line-pairs per picture width, the lens on the larger sensor will produce an image with higher contrast. *It will be sharper.*

Of course, the full-frame image will have less depth of field than the m4/3 image given that they are shot at the same aperture. To equalize depth of field, the m4/3 camera could be shot at f/4. That would also mean that the diffraction blur diameter would be reduced to 3.76 µm, and the relative diffraction blur would be only 37.6% of the line-pair width — exactly the same as for the full-frame camera used at f/8. Thus, when comparing cameras with different sensor formats, *diffraction effects on resolution obey the established rules of camera equivalence.* Note that we are not considering the effects of image sampling by the sensor; but in order for equivalence to hold, it would be necessary for the photosites on the m4/3 sensor to have half the pitch of those on the full-frame sensor — meaning that both sensors have the same number of photosites.

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10 http://www.lensrentals.com/blog/2013/11/otus-is-scharf

5. Discussion

5.1. What Limits Resolution?

As already noted, the achieved resolutions of lenses + sensors are typically less, often much less, than the theoretical limits given in Tables 2 and 3. That begs the frequently debated question of whether lenses or sensors limit resolution in typical cases. For the Zeiss Otus 55mm f/1.4, we won't know until the lens is tested on a sensor with much higher photosite density. The photosite pitch of the D800E is 4.84 µm, or 206.6 photosites per millimeter. That means that the maximum resolution achievable by the sensor is 103 lp/mm (*two* photosites per line-pair). In the previous paper in this series, I demonstrated that it may require a sampling frequency of *four* photosites per line-pair in order to recover the *contrast* of the lens image.[11] If that is the case, the maximum resolution achievable by the D800E sensor, with good contrast, is only 52 lp/mm, which is very nearly the resolution reported for the Otus. That is an argument, although hardly conclusive evidence, for sensor-limited resolution in this particular case.

One other example may reinforce the point that the sensor can play an important role in determining the overall resolution of the imaging system. A 24MP APS-C sensor — state-of-the-art as this is written — has a pixel pitch of about 4 µm. That means that the highest resolution that the sensor can record, regardless of contrast, is 125 lp/mm (two photosites per line-pair). A quick glance at either Table 2 or 3 shows that the theoretical resolution of a perfect lens is considerably greater than 125 lp/mm for many combinations of aperture and contrast ratio. Until we get "large-ish" format sensors with much greater photosite densities, the question of what limits resolution is going to be difficult to answer.

A frequently repeated refrain on internet photography sites is that it requires very good lenses to take full advantage of the 24 and 36MP sensors that are widely available today. I wonder if the opposite is not often the real case — that we need much higher resolution sensors in order to see everything that high quality lenses can deliver.

5.2. Does it Matter?

Today, almost all images, if they are viewed at all, are displayed on smart phone screens, televisions, or typical computer displays. All are fundamentally low-resolution devices in terms of total screen pixels. Given those display modes, almost all current "cameras" are more than good enough. There is little, if anything, to be gained by pushing image resolution closer to its theoretical limits. On the other hand, if the goal is to produce large prints (20 inches or larger), there is an argument for higher resolution images. My current printer is an Epson 3880. The lesser of its native resolutions is 360 ppi, which is equivalent to just over 7 lp/mm. In fact, the Epson can print a very nice high-contrast set of line-pairs at that resolution on glossy paper. They are readily visible at relatively close viewing distance under good light.[12] We require 3,500 line-pairs to fill a 50 cm (19.7 in) wide print at 7 lp/mm. If the image is made with a full-frame camera (36 mm wide), then the necessary resolution of the lens image would be 97.2 lp/mm.

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12 The higher native resolution of the Epson 3880 is 720 ppi, which corresponds to more than 14 lp/mm. It can print line-pairs at that resolution, although a loupe is required to see them.

This is about 75% greater than the MTF50 resolution obtained with the Zeiss Otus 55mm f/1.4 on the Nikon D800E, in the example mentioned above. For a sensor to sample 97.2 lp/mm without seriously degrading contrast, it might be necessary for the photosite density to be 389 per millimeter. That is a photosite pitch of 2.57 µm. The full-frame sensor would have approximately 131 million photosites (3.6 times the number of the D800E).[13,14] For comparison, the photosites of the iPhone 6 are 1.5 µm.

In conclusion, I argue that there is still considerable need for improvement in image resolution if the goal is printing at large sizes, or displaying on large screens with high pixel density.[15] It is certain that improvement will require higher resolution sensors. It is less clear whether, and how much, improvement may be required of lenses; because our current data on lens resolution is confounded with the sensors used to test them.

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13 In order to print the image 19.44 inches (49.4 cm) wide at 360 ppi, it would be necessary to downsample it to 7,000 pixels on the long dimension. Or, it could be printed on the Epson 3880 at 720 ppi at the same size without downsampling.

14 It is possible to use the Otus + D800E to produce a 49.4 cm print with a resolution equivalent to 7 lp/mm. Make a “panorama” by stitching two or more images with 48.6 lp/mm resolution. The stitched image will have a virtual width of 72 mm, will be 14,876 pixels wide, and contain the required 3,500 line-pairs. Resample the image slightly so that it is 14,000 pixels wide and print it at 720 ppi. The 3,500 line-pairs will then be placed on the print at 7 lp/mm. Of course, one could take this approach to its logical conclusion. Stitch a larger number of images so that the virtual camera image is 144 mm wide. This image will contain 7000 line-pairs. Resample the image to 14,000 pixels wide and print at 720 ppi. The print would consist of line-pairs at a frequency of 14 per mm. This seems to be the basic logic behind Ming Thein’s Ultraprints: http://blog.mingthein.com/2014/02/27/introducing-the-ultraprint/

15 The pixel density of the iPhone 6 display is about 326 ppi. The density of the iMac 5K display is about 217 ppi. Once large displays (20 in or larger) reach densities of 300 ppi or more, which seems inevitable, the argument for pushing image resolution closer to the theoretical limits will be even stronger.

Appendix 1

Diffraction Blur

A simulated Airy diffraction pattern is illustrated in Fig. 7, and a three-dimensional graph of the relative intensity is shown in Fig. 8. The Airy *disk* is the central part of the pattern as far as the middle of the first dark ring (the first minimum). It is standard practice in photography to equate the size of the diffraction blur circle with the Airy disk. That is arbitrary. As can be seen from the figures, the intensity of the pattern falls off markedly before the first minimum is

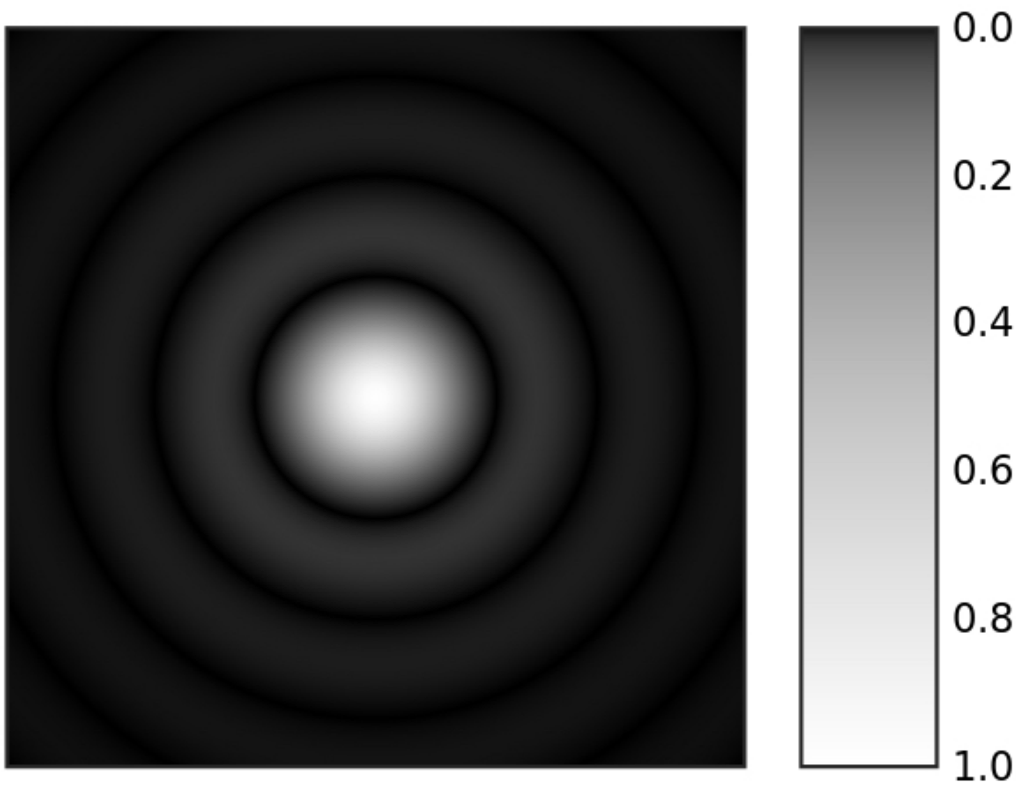

Fig. 7. Simulation of the Airy diffraction pattern. Scale on right indicates relative intensity of light. http://upload.wikimedia.org/wikipedia/commons/thumb/1/14/Airy-pattern.svg/640px-Airy-pattern.svg.png

reached. My tests indicate that the effective size of the diffraction blur circle is about 70% of the diameter of the Airy disk, at which point the intensity of the pattern is only about 11% of its central maximum (Table 4).[16]

[16] http://philservice.typepad.com/f_optimum/2014/05/optimal-aperture-in-photography-3-testing-the-theory-the-diffraction-blur-coefficient.html

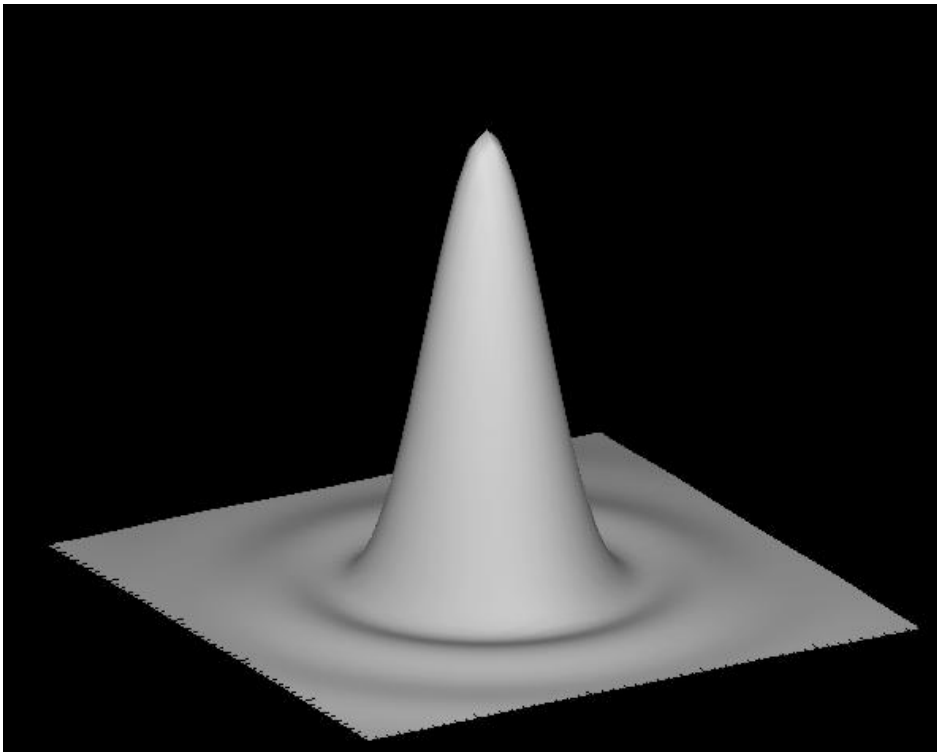

Fig. 8. A three-dimensional representation of the Airy diffraction pattern. Height represents intensity, or brightness. http://www.ctio.noao.edu/~atokovin/tutorial/part1/turb.html

Table 4. Airy Disk Intensity and Distance from Center

Percent of Distance from Center to First Minimum	Relative Brightness, %*
50	36.8
55	29
60	22.1
65	16.1
70	11.1
75	7.2
80	4.2

* Calculated with assistance from the online Bessel function calculator at http://keisan.casio.com/exec/system/1180573474

Appendix 2

Calculation of Blur Circle Overlap on Black Lines

The areas of blur circle overlap on black lines are segments, or partial segments, of circles. In Fig. 4, the segment that overlaps that black line on the right has a height h = 4 µm, and a base that is d = 8.5 µm from the center of the blur circle, which has a radius r = 12.5 µm. A standard formula for the area of a segment is:

$$r^2 \arccos\left(\frac{d}{r}\right) - d\sqrt{2rh - h^2}$$

For this segment, the area is 128.6 - 77.9 = 50.7 µm^2.

The blur circle overlap with the black line on the left is slightly more complicated. The calculation requires determining the areas of two segments. The larger segment is defined by d = 1.5 µm (the distance from the center of the blur circle to the right edge of the black line on the left), and h = 11 µm. This area is 226.6 - 18.6 = 208 µm^2. The smaller segment is defined by the overlap of the blur circle with the white line on the left. The area of this segment must be subtracted from the area of the larger segment in order to obtain the area of overlap on the black line. For the smaller segment, d = 11.5 µm, and h = 1.0 µm. The area is 62.9 - 56.3 = 6.6 µm^2. Therefore, the area of overlap on the black line to the left is 208 - 6.6 = 201.4 µm^2.

Thus, the total black line overlap attributable to this blur circle is 50.7 + 201.4 = 252.1 µm^2, or about 51% of the total area of the circle. This reduces the lightness of the “point” that represents the center of the circle to 0.49. It remains to allocate the lost light to various distances in the overlap areas. This is accomplished by subtraction of nested segments within the overlap zones. For example, the following procedure could be used to calculate the amount of light transferred to the “first” 0.1 µm on the left edge of the black line on the right. We already know from above that the area of the entire right-hand segment is 50.7 µm^2. To find the area of the first 0.1 µm, we need to subtract from 50.7 the area of the segment defined by d = 8.5 + 0.1 = 8.6 µm and h = 4 - 0.1 = 3.9 µm. The latter area is 48.9 µm^2. Thus the area of the first 0.1 µm strip is 50.7 - 48.9 = 1.8 µm^2. This is 0.37% of the total area of the blur circle. Thus the lightness of this region of the black line is increased by 0.0037. The remainder of the light is distributed in a similar fashion by taking differences between smaller and smaller pairs of nested segments in a “walk” from left to right across the overlap area. In actual practice, the “steps” in the walk were much smaller than in this example. A similar procedure is used to allocate light in the black line to the left side of the blur circle.

We do not need to take account of the parts of the blur circle that overlap white lines. There will always be reciprocal and equal exchange of light between every pair of white points with overlapping blur circles.

Many blur circles may overlap a particular region of a black line (Fig. 3). The total light transferred to that region is obtained by summing the contributions of all the overlapping circles. The entire set of calculations can be performed very quickly with a computer program. The data required to generate Fig. 5c, or a line in Table 1, for example, can be obtained in less than one second.

Appendix 3

Comparison With Other Estimates of Resolution Limits

After completing this paper, I came across other estimates of aperture-specific resolution limits. The most complete set is provided by Roger Clark.[17] Clark gives limits for 80% MTF, 50% MTF and for the Rayleigh resolution criterion. The Rayleigh criterion states that images of two point light sources can be distinguished if the maximum of the Airy diffraction pattern produced by one point corresponds to the first minimum of the Airy pattern produced by the second. This means that the maxima of the two Airy patterns are separated by one Airy disk *radius*. The conventional calculation of the Airy disk radius is one-half the value given by Eq. 1. To convert a Rayleigh limit calculation into line-pairs per millimeter, one assumes that the maxima of Airy disks correspond to the centers of white lines. Thus the distance between the centers of two adjacent white lines (i.e., the width of one line pair) is equal to the Airy disk radius. The Rayleigh limit separation of two Airy disks is said to correspond to a modulation transfer function value (MTF) of about 9%. Clark's 80% and 50% MTF resolutions are calculated from the Dawes resolution limit.[18] His limits are reproduced in Table 4, together with results from my simulations.[19]

Table 4. Comparison of Theoretical Resolution Limits (lp/mm)*

Aperture	Rayleigh / 9% Contrast			MTF50 / 50% Contrast			MTF80 / 80% Contrast		
	A	B	C	A	B	C	A	B	C
2	820	420	600	390	293	419	160	231	330
2.8	580	300	428	280	210	299	110	165	236
4	410	210	300	190	147	209	80	116	165
5.6	290	150	214	140	105	150	58	83	118
8	200	105	150	97	73	105	40	58	82
11	150	76	109	71	53	76	29	42	60
16	100	53	75	48	37	52	19	29	41
22	75	38	55	35	27	38	15	21	30

* Data in columns headed by "A" are from Clark. Data for 50% and 80% contrast ratios are from Table 3 ("B" columns) and Table 2 ("C" columns). Resolutions in the "B" columns are for diffraction blur calculated according to Eq. 1; resolutions in the "C" columns are for diffraction calculated according to Eq. 2.

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17 Clark, Roger N. http://www.clarkvision.com/articles/scandetail/#diffraction Cited by Norman Koren here.

18 The Dawes limit is 1/(Nλ) in lp/mm, where N is aperture and λ is wavelength in millimeters.

19 Note that Clark uses λ = 500 nm, rather than 550 nm.

The most appropriate comparisons are between the "A" and "B" columns in each group because both sets of data use the standard formula for calculation of diffraction blur — the diameter of the Airy disk given by Eq. 1. However, a slight complication is that Clark uses λ = 500 nm for his calculations, while I use λ = 550 nm for my simulations. Thus, a more accurate comparison is obtained by reducing the values in the "A" columns by ~10% in order to remove the effect of wavelength differences. Clark's Rayleigh limit resolutions are much greater than my 9% contrast ratio resolutions (using either Eq. 1 or Eq. 2 to calculate diffraction blur). For the MTF50 / 50% contrast case, Clark's numbers are still greater than mine (by a factor of ~1.2) even when diffraction is calculated the same way with the same wavelength. However the pattern is reversed for the MTF80 / 80% contrast case.

Direct calculation of resolution limits based on the Rayleigh or Dawes criteria involve at least one unrealistic assumption: that a white line in an image of line-pairs corresponds to a white line of *infinitesimal* width in the object space.[20] In essence, the formulas are telling us how close white lines of essentially zero width can be (on a black background) and still be resolved. In reality the recognizable objects that we see in typical photographs, including photographs of test charts, are hardly infinitesimal. Based on this observation, I would predict that calculated resolution limits would be higher than limits obtained by simulation, as is the case when contrast is 50% or less (Table 4). It is not clear, however, why the pattern of difference reverses for high contrast (80%). Perhaps there is not a straightforward relationship between MTF values and contrast ratios as calculated here.

[20] The Rayleigh criterion was developed in the context of resolving images of point light sources against a black background, such as images of distant stars surrounded by the black void of space. In that context it is reasonable to consider the light sources to be infinitesimal. The Dawes limit has a similar history in that it was formulated by asking human observers to resolve binary stars.