 

# Limits of Resolution. 1. Sampling Frequency

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## Summary

A camera sensor samples the image that is produced by the lens. The sampling frequency is determined by the photosite pitch of the sensor. In the hypothetical case where a lens is producing images of black and white line-pairs without blur, it is shown that the sampling frequency must be at least four photosites per line-pair in order to recover both the frequency and original contrast of the line-pairs. That is twice the Nyquist sampling rate. Higher sampling frequencies more faithfully reproduce the image formed by the lens. In the case of line-pairs of a given frequency, the edges of the lines become sharper with higher frequency sampling. Published resolution data for the Zeiss 55mm f/1.4 Otus lens on the Nikon D800E are discussed with regard to theoretical limits based on sampling frequency. A case is made that resolution is sensor-limited in that test.

**Key words:** resolution limits, line-pairs, sampling frequency, Nyquist rate, Nikon D800E, Zeiss 55mm f/1.4 Otus Distagon T*, photosite pitch, sensor-limited, lens-limited

## 1. Introduction

The amount and quality of detail that is captured in a digital image is a function of both the sharpness of the lens image *and* the sampling frequency of the sensor. The quality of the lens image (in the plane of focus) will depend on the design and construction of the lens and on aperture. In order to discuss resolution in familiar, quantitative terms, my presentation is based on an image model of black and white line-pairs. I do not claim that resolution in line-pairs per millimeter (lp/mm), together with a specified modulation transfer function value (MTF), is the the best measure of resolution for "real-world" camera use. For my purposes, however, it has the virtue of being relatively easy to model, both graphically and quantitatively.

Here is the way to think about the resolution of lenses on digital cameras. *Lens resolution and sensor resolution interact to determine the achieved resolution of the imaging system.* Given, for example, that we have a camera sensor with a photosite pitch of 4 microns — what is the maximum resolution, in line-pairs per millimeter, at each aperture and at a specified contrast ratio that could be achieved with a "perfect" lens? In the perfect lens scenario, the limits of resolution are determined by photosite pitch and diffraction blur (aperture). Those limits are a metric with which to evaluate the performance of real lenses on actual sensors.

The present paper is a consideration of line-pair sampling in the absence of blur. The model makes it possible to analyze the effect of sampling frequency on resolution and contrast without the confounding effects of diffraction, for example.[1] The effect of blur is taken up in a companion paper.

### 1.1. Terminology

In order to avoid confusion, we need to define a few terms. The *lens image* is the image that is projected by the lens onto the camera sensor. The lens image is an effectively continuous, *analog* representation of the external world in front of the lens. The *sensor image* is the image that is created by processing the light signals recorded by the sensor. The sensor image is a discontinuous, or discrete, *digital sample* of the lens image. Understanding that the sensor *samples* the lens image image is a key step. More samples means that the sensor image will be more representative of the lens image. "More samples" in this case means more photosites on the sensor.[2] The word *frequency* will be used often in this paper. The statement "lens X on camera Y has an MTF50 resolution of 52.4 lp/mm" contains frequency information — in this case we are saying that the frequency of line pairs is 52.4 per millimeter when they are resolved with an MTF value of 50%.[3] The *sampling frequency* of a digital camera sensor is the number of photosites per unit length (usually one millimeter).

## 2. Line Pairs and Sensor Sampling Frequency in the Absence of Blur

### 2.1. Two Photosites per Line-Pair (Nyquist Rate)

Fig. 1a illustrates a lens image of four black and white line-pairs. Because the lens has no aberrations and we are assuming no diffraction blur, the lines in the lens image have perfectly sharp edges and maximum contrast. If we assign a value of 1 to white lines and 0 to black lines, the contrast ratio is (1 - 0) / (1 + 0) = 1, or 100%. In signal-processing terms, the lens image represents the "signal" that we will sample with the sensor. The *frequency* of the signal is 4 line pairs per figure width.[4] Fig. 1b shows a grid of sensor photosites superimposed upon the lens image. Note that the width of a column of photosites is the same as the width of one line (black or white) in the lens image. Thus, the frequency of photosites is 8 per figure width. Therefore, the sampling frequency (imposed by the spacing of photosites) is twice the signal frequency — two photosites per line-pair. This is commonly known as the Nyquist Rate. It is the *minimum*

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[1] I also ignore any complications that might arise from use of a Bayer filter array on the sensor.

[2] This is the real advantage of more camera megapixels, although the advantage is often phrased in terms of being able to record more detail. That is true only to the extent that the "additional" detail is actually present in the lens image to begin with.

[3] There is another meaning of "resolution". It is almost universal to describe the *total number* of photosites ("pixels") on a sensor as its resolution. I do not object to this usage. Consider two sensors, one with 12MP and the other with 24MP. If they are the same physical size, the 24MP sensor must have more photosites per millimeter — that is, greater resolution in the strict sense — than the 12MP sensor. If the two sensors are different sizes but are sampling lens images with the *same field of view*, the 24MP sensor is sampling with greater resolution in terms of *samples per meter of the object field.*

[4] If you prefer to think of frequency in more concrete terms, you can add a dimension to Fig. 1a. Suppose it represents a 0.1 mm wide region of the entire lens image. Then the frequency of line-pairs is 40/mm. For our present purposes, however, we need be concerned only with relative frequencies.

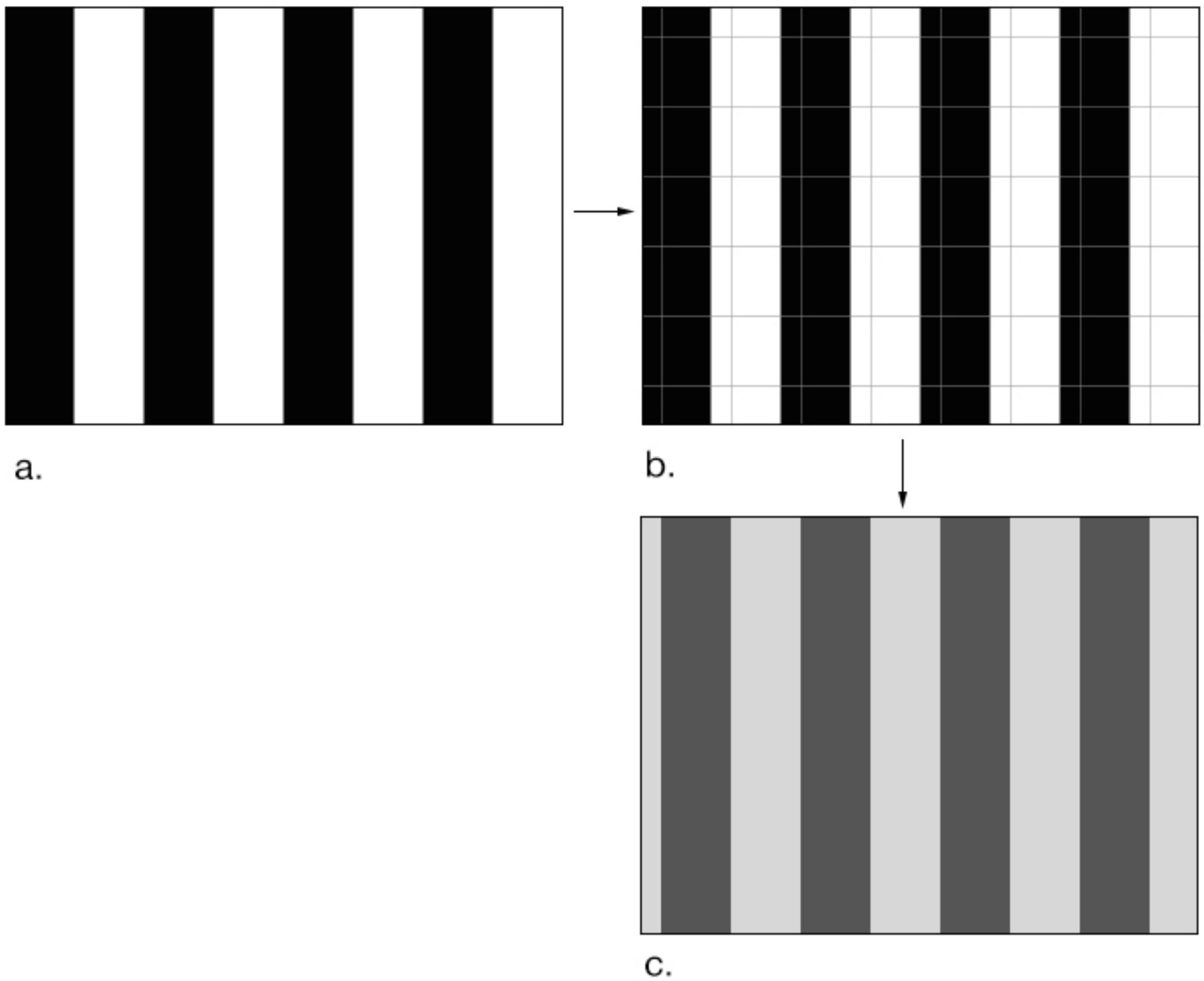



Fig. 1. a. Lens image of four line-pairs without blur. b. Overlay of a grid of photosites that have twice the frequency of the line-pairs. c. Resulting sensor image.

sampling rate that can detect the frequency of line pairs in the signal. Fig. 1c depicts the sensor image that results from the alignment of photosite columns with line-pairs in Fig. 1b. As can be seen, every column of photosites overlaps both white and black lines. Therefore the sensor image will be composed of pairs of uniform light and dark gray lines. The frequency of line-pairs in the sensor image is the same as in the lens image, but we have not recovered the original contrast ratio — the ratio in Fig. 1c is about 43%.

The contrast ratio produced by Nyquist rate sampling is variable. If the photosite columns coincide with the black and white lines of the lens image, the resulting contrast ratio of the sensor image is 100% (Figs. 2a and b). On the other hand, if the alignment is such that the photosite columns are exactly one-half photosite out of phase with the lens-image line-pairs, the resulting contrast ratio is zero (Figs. 2c and d). Given that alignment will normally be haphazard, the average contrast ratio with Nyquist rate sampling is 50%, assuming that there is

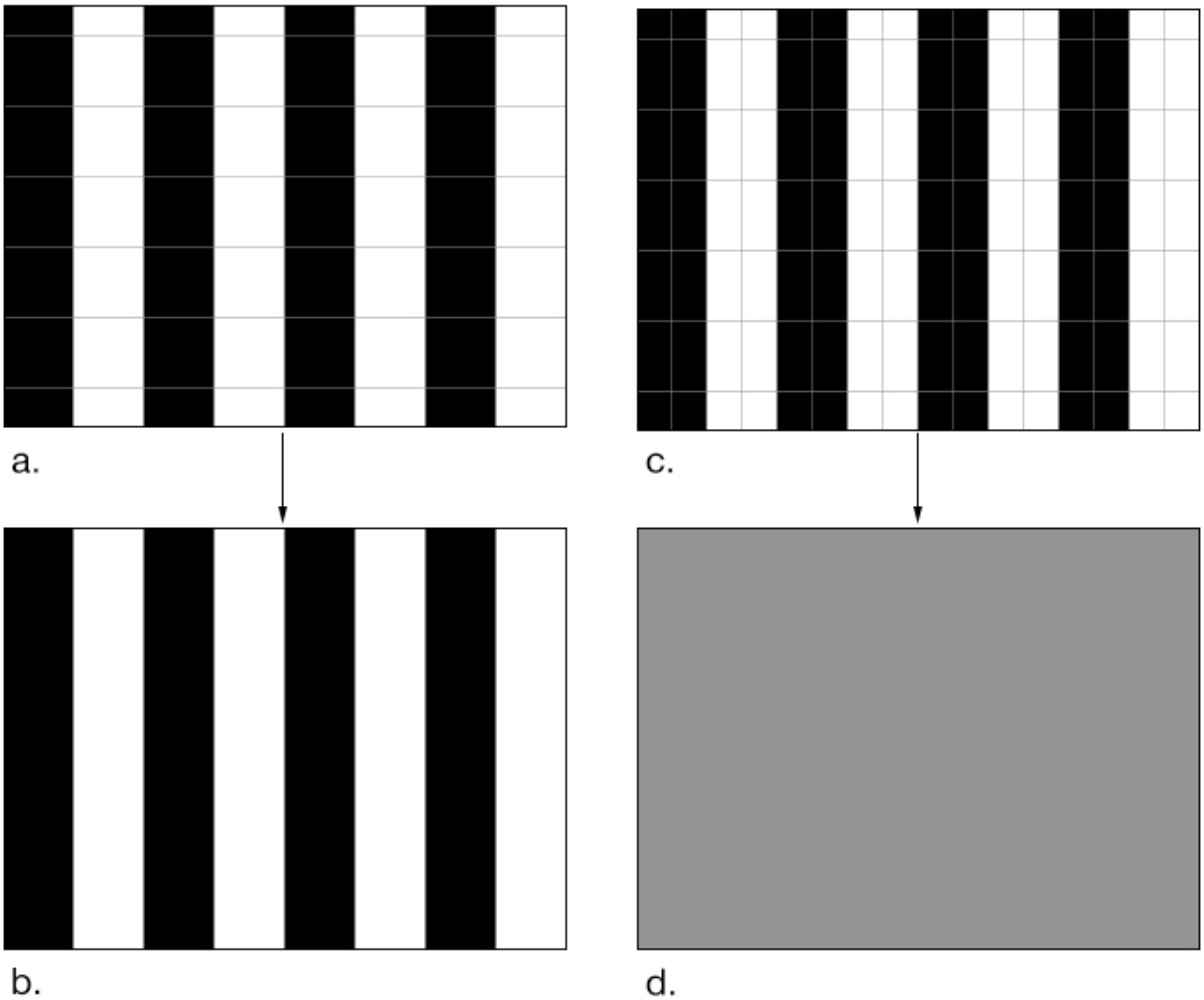



Fig. 2. Effect of photosite grid alignment when photosite frequency is two per line-pair. a. Grid perfectly aligned with line-pairs. b. Sensor image corresponding to Fig. a. c. Grid spacing one-half photosite out of phase with line-pairs. d. Sensor image corresponding to Fig. c.

no blur. In summary, Nyquist rate sampling of line-pairs is sufficient to recovery the frequency of the signal, but not its amplitude (contrast ratio).

### 2.2. One and One-Half Photosites per Line-Pair (0.75x Nyquist Rate)

Figs. 3a and b show what happens if the photosite pitch is such that frequency of photosites is less than twice the frequency of lens-image line-pairs. In this example, the sampling frequency is 1.5 photosites per line-pair (0.75x Nyquist rate). The resulting sensor image consists of alternating light, medium, and dark gray lines. The frequency of lines of any particular shade is only about two per figure width. Clearly, we have lost both the frequency and amplitude of the original signal. Less than Nyquist rate sampling will not be considered further.

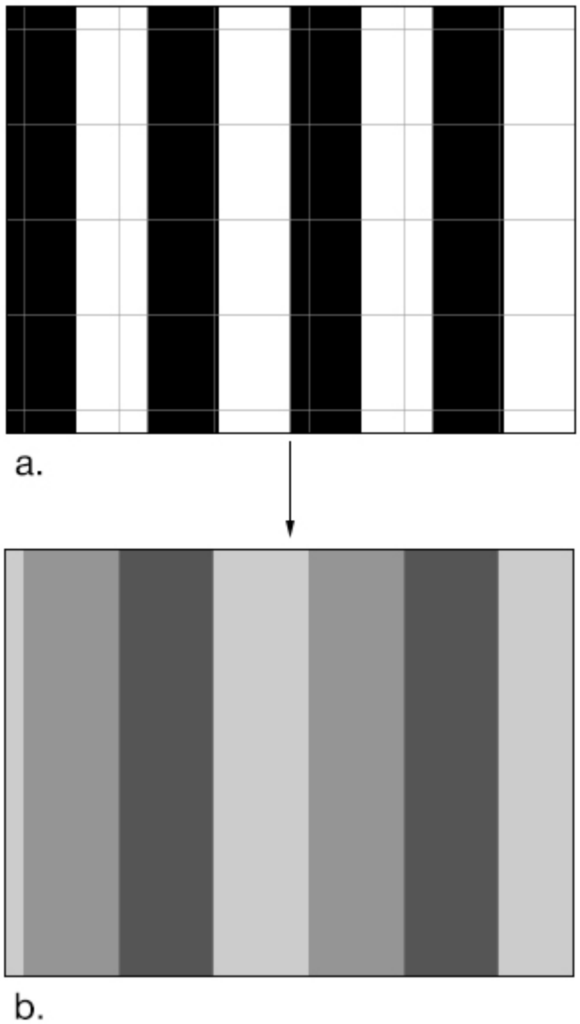



Fig. 3. Effect of sampling with 1.5 photosites per line-pair. a. Photosite grid overlay on lens image. b. Resulting sensor image.

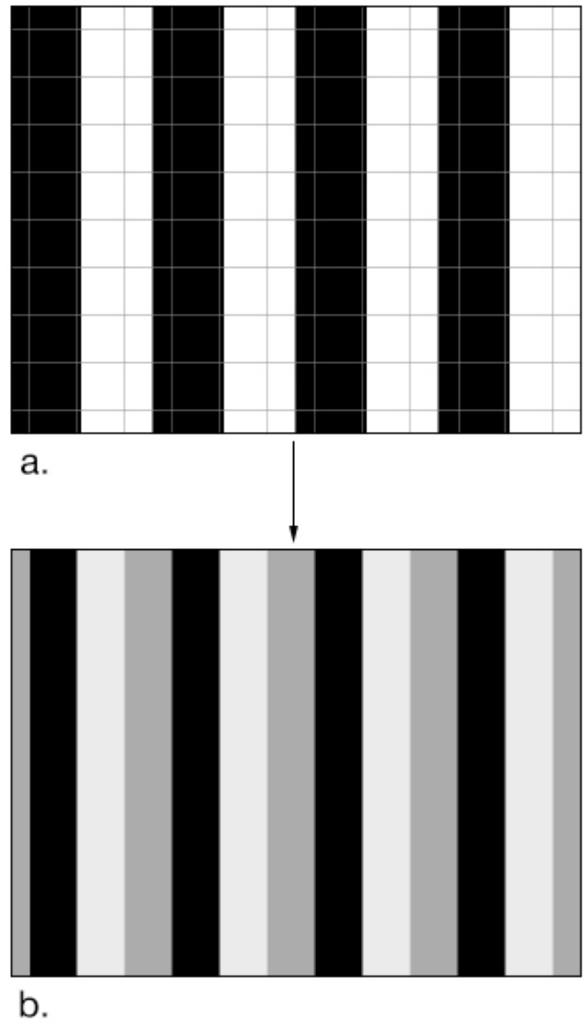


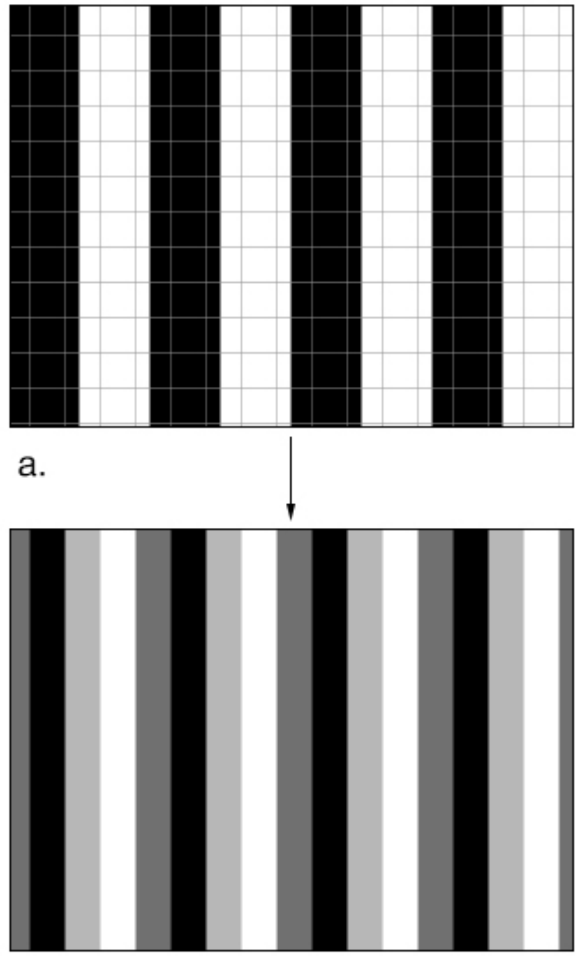

Fig. 4. Effect of sampling with three photosites per line-pair. a. Photosite grid overlay on lens image. b. Resulting sensor image.

### 2.3. Three Photosites per Line-Pair (1.5x Nyquist Rate)

For recording images of line-pairs, the most interesting and useful cases involve sampling at greater than Nyquist rate. Let's begin by seeing what happens when the frequency of photosites is 3x the frequency of line pairs (1.5x Nyquist rate). The sensor image consists of repeating triplets of lines (Fig. 4b). There are four triplets across the figure, so the frequency of the underlying signal has been recovered. The contrast ratio between the lightest and darkest lines will depend upon the alignment of the photosite grid with the black and white lines of the lens image (Fig. 4a). In this particular case, the maximum contrast ratio is about 92%. Therefore, the amplitude of the underlying signal (its contrast ratio) has also been almost completely recovered. However some information is still missing — it is difficult to interpret the repeating pattern of lines in Fig. 4b as *line-pairs,* although we do have *line-triplets* at the correct frequency. We need a sampling frequency that is at least twice the Nyquist rate (4x the frequency of line-pairs) in order to produce a sensor image that resembles repeating line-pairs.

### 2.4. Four Photosites per Line-Pair and Higher (2x Nyquist Rate and Above)

Sampling line-pairs at four photosites per line-pair (twice Nyquist rate) yields a recognizable pattern of black and white lines with gray "borders" (Figs. 5a and b). At this sampling rate we recover both the frequency and amplitude (contrast ratio) of the original analog lens image, albeit with gray lines interposed between the black and white lines. In other words, sampling at a rate of four photosites per line-pair is sufficient to extract all the "information" in the lens image — in the absence of blur. The gray lines are one photosite wide, and always will be except in the unlikely event that the photosite grid coincides exactly with the line-pairs in the lens image. It follows that the effect of increasing the sampling rate — more photosites per line-pair — will be to make the gray "borders" narrower, but will not yield any more resolution (lp/mm) or contrast (based on the difference between lightest and darkest lines). The effect, therefore, of increasing the photosite density beyond twice Nyquist is to make the sensor image a more faithful representation of the lens image by sharpening line edges. This is the sense in which more photosites are always better, even if there is no more "resolution" to be gained from the lens image.[5]



a.

b.

Fig. 5. Effect of sampling with four photosites per line-pair. a. Photosite grid overlay on lens image. b. Resulting sensor image.

## 3. An Example

Consider a camera sensor with a 5 µm photosite pitch. The sensor samples the lens image at the rate of (1 / 0.005) = 200 per millimeter. If 200/mm represents the Nyquist rate, then

5 Here and elsewhere, any statement to the effect that "more photosites are always better" implies "all other things being equal." Obviously, if increasing the number of photosites makes them smaller, then noise characteristics, dynamic range, ISO, etc., are likely to be affected.

the sensor should be able to detect line pairs at half that frequency, or 100 lp/mm. *We have, therefore, a fundamental limit — 100 lp/mm is the highest line-pair frequency that can be recorded by a sensor with a 5 µm photosite pitch. It is immaterial if the the lens is capable of resolving line-pairs at a higher frequency. Furthermore, we know that the expected contrast ratio will be 50%,* ***in the absence of blur.*** *Given the unavoidability of diffraction blur, the best that our sensor could possibly do in practice would be 100 lp/mm with contrast considerably less than 50%.*

What if 200 photosites/mm represents twice Nyquist rate? That means sampling at four photosites per line-pair, and the corresponding line-pair frequency would be 50 lp/mm. From section 2.4, in the absence of blur, we would expect the contrast ratio of the sensor image to be 100%. *Here, then is another fundamental limit. In order for a sensor with 5µm photosite spacing to record black and white line-pairs with 100% contrast — eqivalently MTF100 — the line-pair frequency can be no greater than 50/mm.* Again, in any real case, there will be blur. Therefore, we would expect that any report of 50 lp/mm resolution by this sensor would also entail an MTF value considerably less than 100%.

It follows that if we wanted to be able to record sensor images with 100 lp/mm resolution and a contrast ratio of 1.0, we would need a sensor with at least 400 photosites per millimeter — a photosite pitch of 2.5 µm. Yes, more photosites does mean more resolution, *if the detail is in the lens image.*

### 3.1. The Zeiss 55mm f/1.4 Otus Lens and the Nikon D800E — Lens-Limited or Sensor-Limited Resolution?

As of this writing (mid-2014), the Zeiss 55mm f/1.4 Otus is widely considered to be the sharpest available "full-frame", normal focal length, prime lens. It is instructive to consider the performance of that lens in the present context. MTF50 resolution has been reported by LensRentals.com,[6] with the lens mounted on a Nikon D800E. LenRentals uses Imatest to determine resolution, and reports results as line-pairs per picture height. 24mm is the nominal "height" of a full-frame sensor in landscape orientation. The lens achieves maximum MTF50 resolution of 55.6 lp/mm at f/5.6. The photosite frequency of the D800E sensor is approximately 205/mm. Thus, maximum MTF50 resolution is achieved when the sampling rate by photosites is (205 / 55.6) = 3.69x the line-pair frequency. That is remarkably close to the 4 photosites per line-pair that are needed to record line pairs with 100% contrast in the absence of blur (Sec. 2.4). Given that there must at least be diffraction blur in the LensRentals test, it is possible that the observed MTF50 resolution is the best that could be achieved with the photosite spacing of the D800E sensor. In other words, *the resolution of the 55mm Otus may well be sensor limited in this test.* We won't know unless the Otus is tested on a sensor with more photosites per millimeter.

[6] http://www.lensrentals.com/blog/2013/11/otus-is-scharf The resolution at f/4 is very similar: 55.0 lp/mm.

## 4. Conclusion

Resolution, in line-pairs per distance, that can be achieved with a digital sensor is strongly constrained by the photosite frequency of the sensor. Even in the absence of blur from diffraction and lens aberrations, the highest line-pair frequency than can be obtained with high contrast will be only about one-fourth the photosite frequency. Therefore, resolution results reported in lens tests should be evaluated in light of the photosite frequency of the camera sensor. Maximum MTF50 resolutions that are substantially less than one-fourth the photosite frequency are suggestive of lens-limited systems. On the other hand, a maximum MTF50 resolution that is about one-fourth the photosite frequency is suggestive of a sensor-limited system. In order to refine the analysis, it is necessary to consider the effects of diffraction on contrast.